PACE

Policy Analysis based on Computable Equilibrium

I. Basic model description

II. Dynamic extension

1. Basic model description

1.1 Non-technical Model Summary

Figure 1 provides a diagrammatic structure of the comparative-static multi-region, multi-sector CGE model PACE. Primary factors of region r include labor $\overline{L}_r$, capital $\overline{K}_r$, and fossil-fuel resources $\overline{Q}_{ff,r}$. Labor and capital are intersectorally mobile within a region but cannot move between regions. A specific resource is used in the production of fossil fuels *ff* (crude oil, coal and gas), resulting in upward sloping supply schedules.

Production Y_{ir} of commodities i in region r other than primary fossil fuels is captured by aggregate production functions which characterize technology through substitution possibilities between various inputs. Nested constant elasticity of substitution (CES) cost functions with three levels are employed to specify the substitution possibilities in domestic production between capital, labor, energy and non-energy, intermediate inputs, i.e. material. At the top level, non-energy inputs are employed in fixed proportions with an aggregate of energy, capital and labor. At the second level, a CES function describes the substitution possibilities between the energy aggregate and the aggregate of labor and capital. Finally, at the third level, capital and labor trade off with a constant elasticity of substitution. As to the formation of the energy aggregate, we allow sufficient levels of nesting to permit substitution between primary energy types, as well as substitution between a primary energy composite and secondary energy, i.e. electricity.

Final demand C_r in each region is determined by a representative agent RA_r, who maximizes utility subject to a budget constraint with fixed investment. Total income of the representative household consists of factor income and tax revenues. Final demand of the representative agent is given as a CES composite which combines consumption of an energy aggregate with a non-energy consumption bundle. Substitution patterns within the non-energy consumption bundle are reflected via Cobb-Douglas functions. The energy aggregate in final demand consists of the various energy goods trading off at a constant elasticity of substitution.

All goods used on the domestic market in intermediate and final demand correspond to a CES composite A_{ir} of the domestically produced variety and a CES import aggregate M_{ir} of the same variety from the other regions (the so-called Armington good – see Armington, 1969). Domestic production either enters the formation of the Armington good or is exported to satisfy the import demand of other regions.

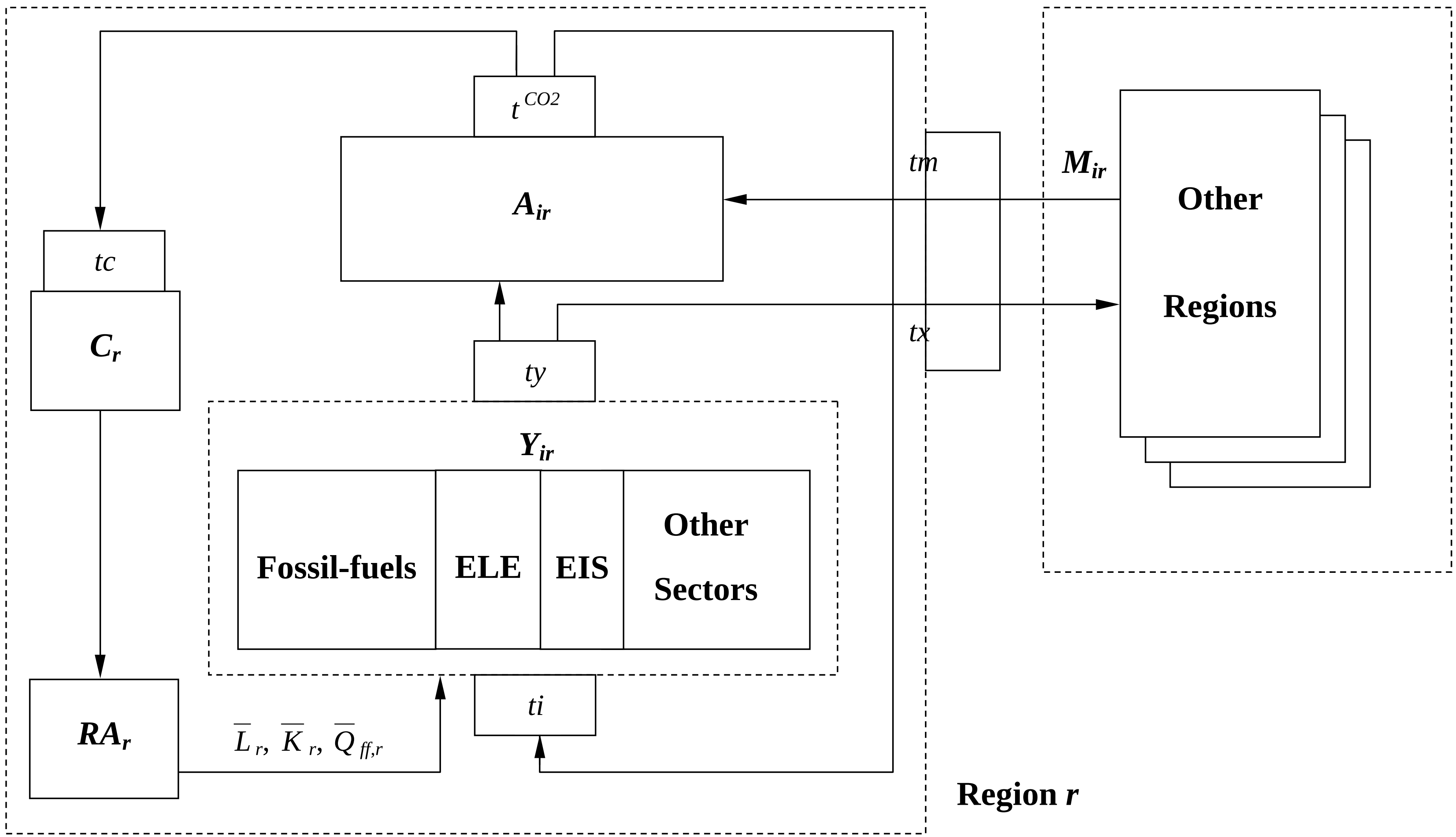
The tax system includes all types of indirect taxes (production taxes or subsidies *ty*, intermediate taxes *ti*, consumption taxes *tc*, as well as tariffs *tm* and *tx*) which are used to finance a fixed level of public good provision. A lump-sum tax on the representative household balances the public budget.

Benchmark data determine parameters of the functional forms from a given set of benchmark quantities, prices, and elasticities. The underlying data base is GTAP-EG for the year 1997 which provides a consistent representation of energy markets in physical units as well as detailed accounts of regional production and consumption as well as bilateral trade flow (see McDougall et al., 1998; Rutherford and Paltsev, 2000). The benchmark data, and the regional and sectoral aggregation are described in section *Benchmark Data - Regional and Sectoral Aggregation* of this Appendix.

The economic effects of future climate policies depend on the extent to which emission reduction targets constrain the respective economies in their *BaU* development (without emission limits). Thus, the magnitude and distribution of adjustment costs to Post-Kyoto commitments depend on the *BaU* projections for GDP, fuel prices, energy efficiency improvements, etc. In our comparative-static framework, we infer the *BaU* structure of the model's regions for the target year (in our case: 2020) using recent projections for economic development from the International Energy Outlook (DOE, 2001) (see section *Baseline Projections - Forward Calibration* of this Appendix). We then measure the costs of abatement relative to that baseline.

Numerically, the model is formulated as a mixed complementarity problem (MCP) in GAMS (Brooke et al. 1996; Rutherford, 1999) and solved using PATH (Dirkse and Ferris, 1995).

Figure 1: Diagrammatic model structure



1.2 Algebraic Model Description

Two classes of conditions characterize the competitive equilibrium for our model: zero profit conditions and market clearance conditions. The former class determines activity levels and the latter determines price levels. In our algebraic exposition, the notation Π_{ir}^{z} is used to denote the profit function of sector *j* in region *r* where *z* is the name assigned to the associated production activity. Differentiating the profit function with respect to input and output prices provides compensated demand and supply coefficients (Hotelling's lemma), which appear subsequently in the market clearance conditions.

We use *i* (aliased with *j*) as an index for commodities (sectors) and *r* (aliased with *s*) as an index for regions. The label *EG* represents the set of energy goods and the label *FF* denotes the subset of fossil fuels. Tables 1 – 6 explain the notations for variables and parameters employed within our algebraic exposition. Figures 2 to 5 provide a graphical exposition of the production and final consumption structure.

1.2.1 Zero Profit Conditions

1. Production of goods except fossil fuels:

$$\prod_{ir}^{Y} = \left(\theta_{ir}^{X} p_{ir}^{X^{1-\eta}} + (1-\theta_{ir}^{X}) p_{ir}^{1-\eta}\right)^{\frac{1}{1-\eta}} - \sum_{j \notin EG} \theta_{jir} p_{jr}^{A} - \theta_{ir}^{KLE} \left[\theta_{ir}^{E} p_{ir}^{E\ 1-\sigma_{KLE}} + (1-\theta_{ir}^{E}) \left(w_r^{\alpha_{jr}^{L}} v_r^{\alpha_{jr}^{K}}\right)^{1-\sigma_{KLE}}\right]^{\frac{1}{1-\sigma_{KLE}}} = 0 \qquad i \notin \mathrm{FF}$$

2. Production of fossil fuels:

$$\prod_{ir}^{Y} = \left(\theta_{ir}^{X} p_{ir}^{X^{1-\eta}} + (1-\theta_{ir}^{X}) p_{ir}^{1-\eta}\right)^{\frac{1}{1-\eta}} - \left[\theta_{ir}^{Q} q_{ir}^{1-\sigma_{Q,i}} + (1-\theta_{ir}^{Q}) \left(\theta_{Lir}^{FF} w_r + \theta_{Kir}^{FF} v_r + \sum_{j} \theta_{jir}^{FF} p_{jr}^{A}\right)^{1-\sigma_{Q,i}}\right]^{\frac{1}{1-\sigma_{Q,i}}} = 0 \qquad i \in \mathrm{FF}$$

3. Sector-specific energy aggregate:

$$\prod_{ir}^{E} = p_{ir}^{E} - \left\{\theta_{ir}^{ELE} p_{\{ELE,r\}}^{A^{1-\sigma_{ELE}}} + (1-\theta_{ir}^{ELE}) \left[\theta_{ir}^{COA} p_{\{COA,r\}}^{A^{1-\sigma_{COA}}} + (1-\theta_{ir}^{COA}) \left(\prod_{j \in LQ} p_{jr}^{A^{\beta_{jir}}}\right)^{1-\sigma_{COA}}\right]^{\frac{1-\sigma_{ELE}}{1-\sigma_{COA}}}\right\}^{\frac{1}{1-\sigma_{ELE}}} = 0$$

4. Armington aggregate:

$$\prod_{ir}^{A} = p_{ir}^{A} - \left[\left(\theta_{ir}^{A} p_{ir}^{1-\sigma_{A}} + (1-\theta_{ir}^{A}) p_{ir}^{M^{1-\sigma_{A}}}\right)^{\frac{1}{1-\sigma_{A}}} + t_r^{CO2} a_i^{CO2}\right] = 0$$

5. Aggregate imports across import regions:

$$\prod_{ir}^{M} = p_{ir}^{M} - \left(\sum_{s} \theta_{isr}^{M} \; p_{is}^{X\;1-\sigma_M} \right)^{\frac{1}{1-\sigma_M}} = 0$$

6. Household consumption demand:

$$\prod_{r}^{C} = p_{r}^{C} - \left(\theta_{Cr}^{E} \; p_{Cr}^{E\;1-\sigma_{EC}} + (1-\theta_{Cr}^{E}) \left[\prod_{i \notin FF} p_{ir}^{A^{\gamma_{ir}}} \right]^{1-\sigma_{EC}} \right)^{\frac{1}{1-\sigma_{EC}}} = 0$$

7. Household energy demand:

$$\prod_{Cr}^{E} = p_{Cr}^{E} - \left[\sum_{i \in FF} \theta_{iCr}^{E} \; p_{ir}^{A\;1-\sigma_{FF,C}} \right]^{\frac{1}{1-\sigma_{FF,C}}} = 0$$

1.2.2 Market Clearance Conditions

8. Labor:

$$\overline{L}_r = \sum_i Y_{ir} \frac{\partial \Pi_{ir}^{Y}}{\partial w_r}$$

9. Capital:

$$\overline{K}_r = \sum_i Y_{ir} \frac{\partial \Pi_{ir}^{Y}}{\partial v_r}$$

10. Natural resources:

$$\overline{Q}_{ir} = Y_{ir} \frac{\partial \Pi_{ir}^{Y}}{\partial q_{ir}} \qquad i \in FF$$

11. Output for domestic markets:

$$Y_{ir} \frac{\partial \Pi_{ir}^{Y}}{\partial p_{ir}} = \sum_j A_{jr} \frac{\partial \Pi_{jr}^{A}}{\partial p_{ir}}$$

12. Output for export markets:

$$Y_{ir} \frac{\partial \Pi_{ir}^{Y}}{\partial p_{ir}^{X}} = \sum_s M_{is} \frac{\partial \Pi_{is}^{M}}{\partial p_{ir}^{X}}$$

13. Sector specific energy aggregate:

$$E_{ir} = Y_{ir} \frac{\partial \Pi_{ir}^{Y}}{\partial p_{ir}^{E}}$$

14. Import aggregate:

$$M_{ir} = A_{ir} \frac{\partial \Pi_{ir}^{A}}{\partial p_{ir}^{M}}$$

15. Armington aggregate:

$$A_{ir} = \sum_{j} Y_{jr} \frac{\partial \Pi_{jr}^{Y}}{\partial p_{ir}^{A}} + C_{r} \frac{\partial \Pi_{r}^{C}}{\partial p_{ir}^{A}}$$

16. Household consumption:

$$C_{r} p_{r}^{C} = w_{r} \overline{L}_{r} + v_{r} \overline{K}_{r} + \sum_{j \in FF} q_{jr} \overline{Q}_{jr} + t_{r}^{CO2} \overline{CO2}_{r} + p_{CGD,r} \overline{Y}_{CGD,r} + \overline{B}_{r}$$

17. Aggregate household energy consumption:

$$E_{Cr} = C_{r} \frac{\partial \Pi_{r}^{C}}{\partial p_{Cr}^{E}}$$

18. Carbon emissions:

$$\overline{CO2}_{r} = \sum_{i} A_{ir} a_{i}^{CO2}$$

Table 1: Sets

I	Sectors and goods
J	Aliased with i
R	Regions
S	Aliased with r
EG	All energy goods: Coal, crude oil, refined oil, gas and electricity
FF	Primary fossil fuels: Coal, crude oil and gas
LQ	Liquid fuels: Crude oil and gas

Table 2: Activity variables

Y_{ir}	Production in sector *i* and region *r*
E_{ir}	Aggregate energy input in sector *i* and region *r*
M_{ir}	Aggregate imports of good *i* and region *r*
A_{dir}	Armington aggregate for demand category *d* of good *i* in region *r*
C_{r}	Aggregate household consumption in region *r*
E_{Cr}	Aggregate household energy consumption in region *r*

Table 3: Price variables

p_{ir}	Output price of good *i* produced in region *r* for domestic market
p_{ir}^{X}	Output price of good *i* produced in region *r* for export market
p_{ir}^{E}	Price of aggregate energy in sector *i* and region *r*
p_{ir}^{M}	Import price aggregate for good *i* imported to region *r*
p_{ir}^{A}	Price of Armington good *i* in region *r*
p_{r}^{C}	Price of aggregate household consumption in region *r*
p_{Cr}^{E}	Price of aggregate household energy consumption in region *r*
w_r	Wage rate in region *r*
v_r	Price of capital services in region *r*
q_{ir}	Rent to natural resources in region *r* (i $\in$ FF)
t_r^{CO2}	CO_2 tax in region *r*

Table 4: Endowments and emissions coefficients

$\overline{L}_r$	Aggregate labor endowment for region r
$\overline{K}_r$	Aggregate capital endowment for region r
$\overline{Q}_{ir}$	Endowment of natural resource i for region r (i$\in$FF)
$\overline{B}_r$	Balance of payment deficit or surplus in region r (note: $\sum_r \overline{B}_r = 0$)
$\overline{CO_2}_r$	Endowment of carbon emission rights in region r
$a_i^{CO_2}$	Carbon emissions coefficient for fossil fuel i (i$\in$FF)

Table 5: Cost shares

θ_{ir}^{X}	Share of exports in sector *i* and region *r*
θ_{jir}	Share of intermediate good *j* in sector *i* and region *r* (i$\notin$FF)
θ_{ir}^{KLE}	Share of KLE aggregate in sector *i* and region *r* (i$\notin$FF)
θ_{ir}^{E}	Share of energy in the KLE aggregate of sector *i* and region *r* (i$\notin$FF)
α_{ir}^{T}	Share of labor (*T*=*L*) or capital (*T*=*K*) in sector *i* and region *r* (i$\notin$FF)
θ_{ir}^{Q}	Share of natural resources in sector *i* of region *r* (i$\in$FF)
θ_{Tir}^{FF}	Share of good *i* (*T*=*i*) or labor (*T*=*L*) or capital (*T*=*K*) in sector *i* and region *r* (i$\in$FF)
θ_{ir}^{COA}	Share of coal in fossil fuel demand by sector *i* in region *r* (i$\notin$FF)
θ_{ir}^{ELE}	Share of electricity in energy demand by sector *i* in region *r*
β_{jir}	Share of liquid fossil fuel *j* in energy demand by sector *i* in region *r* (i$\notin$FF, j$\in$LQ)
θ_{isr}^{M}	Share of imports of good *i* from region *s* to region *r*
θ_{ir}^{A}	Share of domestic variety in Armington good *i* of region *r*
θ_{Cr}^{E}	Share of fossil fuel composite in aggregate household consumption in region *r*
γ_{ir}	Share of non-energy good *i* in non-energy household consumption demand in region *r*
θ_{iCr}^{E}	Share of fossil fuel *i* in household energy consumption in region *r*

Table 6: Elasticities

η	Transformation between production for the domestic market and production for the export	2
σ_{KLE}	Substitution between energy and value-added in production (except fossil fuels)	0.8
$\sigma_{Q,i}$	Substitution between natural resources and other inputs in fossil fuel production calibrated consistently to exogenous supply elasticities μ_{FF}	μ_{COA}=0.5 μ_{CRU}=1.0 μ_{GAS} =1.0
σ_{ELE}	Substitution between electricity and the fossil fuel aggregate in production	0.3
σ_{COA}	Substitution between coal and the liquid fossil fuel composite in production	0.5
σ_{A}	Substitution between the import aggregate and the domestic input	4
σ_{M}	Substitution between imports from different regions	8
σ_{EC}	Substitution between the fossil fuel composite and the non-fossil fuel consumption aggregate in household consumption	0.8
$\sigma_{FF,C}$	Substitution between fossil fuels in household fossil energy consumption	0.3

Figure 2: Nesting in non-fossil fuel production

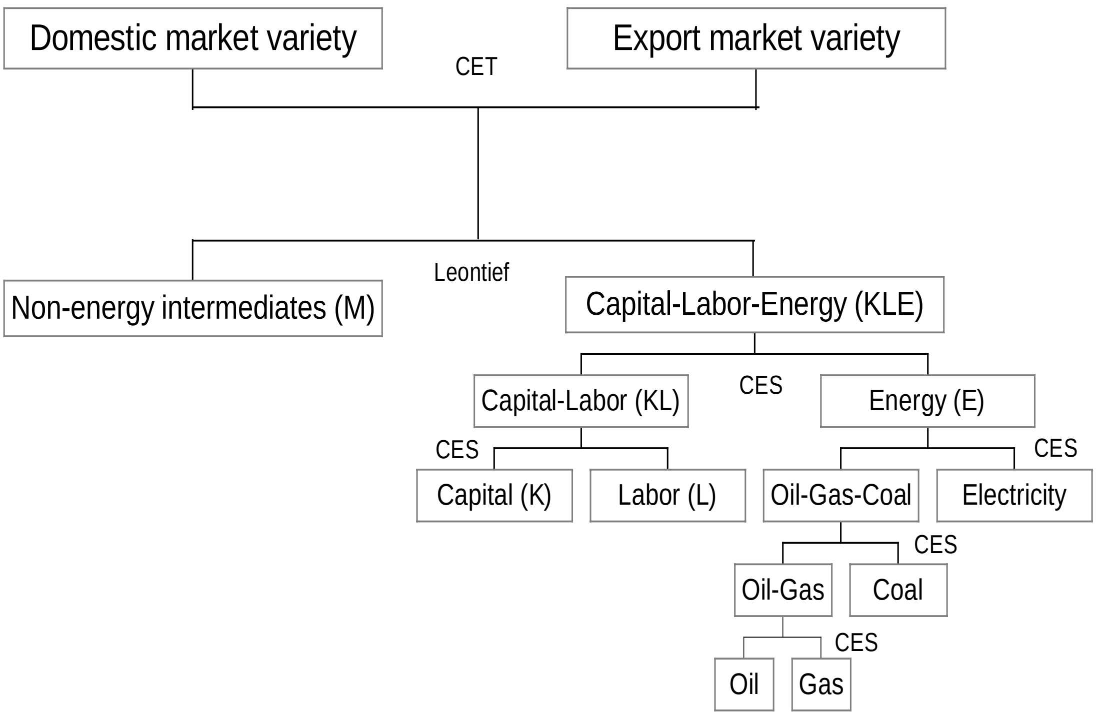


Figure 3: Nesting in fossil fuel production

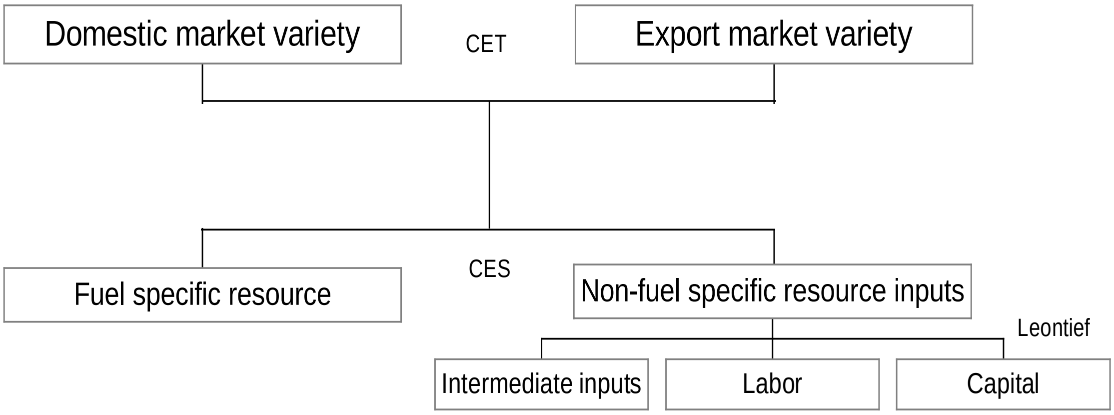


Figure 4: Nesting in household consumption

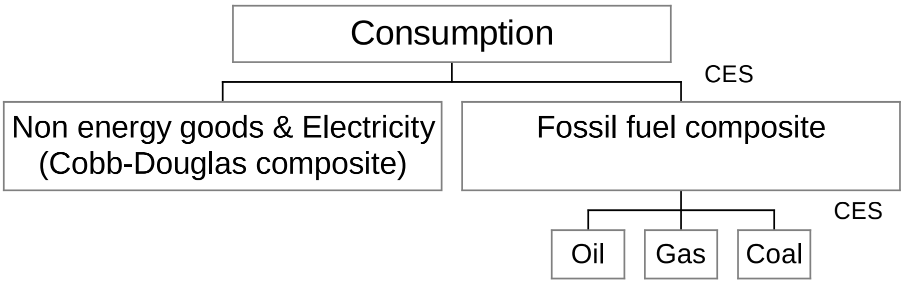
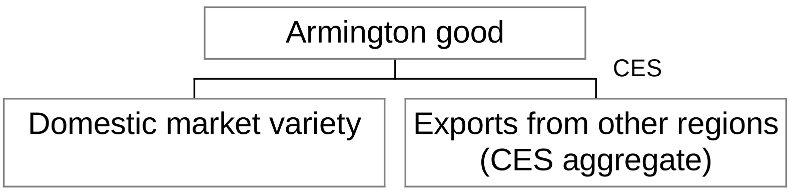


Figure 5: Nesting in Armington production



1.2.3 Benchmark Data - Regional and Sectoral Aggregation

The model is built on a comprehensive energy-economy dataset that accommodates a consistent representation of energy markets in physical units as well as detailed accounts of regional production and bilateral trade flow. The underlying data base is GTAP-EG which reconciles the most recent GTAP economic production and trade dataset for the year 1997 with OECD/IEA energy statistics for 50 regions and 23 sectors (Rutherford and Paltsev, 2000). Benchmark data determine parameters of the functional forms from a given set of benchmark quantities, prices, and elasticities. Sectors and regions of the original GTAP-EG data set are aggregated according to Tables 7 and 8 to yield the model's sectors and regions (see Table 3).

Table 7: Sectoral aggregation

Sectors in GTAP-EG			
AGR	Agricultural products	NFM	Non-ferrous metals
CNS	Construction	NMM	Non-metallic minerals
COL	Coal	OIL	Refined oil products
CRP	Chemical industry	OME	Other machinery
CRU	Crude oil	OMF	Other manufacturing
DWE	Dwellings	OMN	Mining
ELE	Electricity and heat	PPP	Paper-pulp-print
FPR	Food products	SER	Commercial and public services
GAS	Natural gas works	T_T	Trade margins
I_S	Iron and steel industry	TRN	Transport equipment
LUM	Wood and wood-products	TWL	Textiles-wearing apparel-leather

Mapping from aggregate model sectors to GTAP-EG sectors*		
Energy		
COL	Coal	COL
CRU	Crude oil	CRU
GAS	Natural gas	GAS
OIL	Refined oil products	OIL
ELE	Electricity	ELE
Non-Energy		
EIS	Energy-intensive sectors	CRP, I_S, NFM, NMM, PPP, TRN
ROI	Rest of industry	AGR, CNS, DWE, FPR, LUM, OME, OMF, OMN, SER, T_T, TWL

* Set *i* in Table 1 includes two additional artificial production sectors (CGD and G) that denote the (exogenous) demand for an investment/savings good (CGD) and the public good (G).

Table 8: Regional aggregation

Regions in GTAP-EG			
ARG	Argentina	MYS	Malaysia
AUS	Australia	NZL	New Zealand
BRA	Brazil	PHL	Philippines
CAM	Central America & Caribbean	RAP	Rest of Andean Pact
CAN	Canada	RAS	Rest of South Asia
CEA	Central European Associates	REU	Rest of EU
CHL	Chile	RME	Rest of Middle East
CHN	China	RNF	Rest of North Africa
COL	Columbia	ROW	Rest of World
DEU	Germany	RSA	Rest of South Africa
DNK	Denmark	RSM	Rest of South America
EFT	European Free Trade Area	RSS	Rest of South-Saharan Africa
FIN	Finland	SAF	South Africa
FSU	Former Soviet Union	SGP	Singapore
GBR	United Kingdom	SWE	Sweden
HKG	Hong Kong	THA	Thailand
IDN	Indonesia	TUR	Turkey
IND	India	TWN	Taiwan
JPN	Japan	URY	Uruguay
KOR	Republic of Korea	USA	United States of America
LKA	Sri Lanka	VEN	Venezuela
MAR	Morocco	VNM	Vietnam
MEX	Mexico		

Table 8: continued

Mapping from aggregate model regions to GTAP-EG regions		
Industrialized world		
AUN	Australia, New Zealand	AUS, NZL
CAN	Canada	CAN
EUR	OECD Europe (incl. EFTA) and Central and Eastern Associates	CEA, DEU, DNK, EFT, FIN, GBR, REU, SWE, TUR
FSU	Former Soviet Union	FSU
JPN	Japan	JPN
USA	United States	USA
Developing world		
AFR		MAR, RSA, RSS, SAF
ASI		KOR, LKA, PHL, RAS, ROW, SGP, THA, TWN, VNM
CHN		CHN
IND		IND
MPC		IDN, MEX, MYS, RME, RNF, VEN
MSA		ARG, BRA, CAM, CHL, COL, RAP, RSM

1.2.4 Baseline Projections - Forward Calibration

The magnitude and distribution of abatement costs associated with the implementation of the Kyoto emission constraints crucially depend on the *BaU* projections for GDP, fuel prices, energy efficiency improvements, etc. In our comparative-static framework, we infer the *BaU* economic structure of the model's regions for the year 2020 using most recent projections by the International Energy Outlook (DOE, 2001) for GDP growth, fossil fuel production, and future energy prices. We incorporate autonomous energy efficiency improvement factors which scale energy demand functions to match the exogenous emission forecasts. The concrete forward calibration of the model entails three steps.

First, we fix the time profile of fossil fuel supplies from the model's regions to the exogenous baseline projections by making supplies inelastic and scaling sector-specific resources with the exogenous growth rates in fossil fuel production. This allows us to partially control the emission profile from the supply side. Within the *BaU* calculation, we endogenously adjust the resource endowments of fossil fuels to calibrate the model to given exogenous target prices for fossil fuels. At the same time we incorporate exogenous, region-specific GDP growth rates to scale the labor and capital stock of our static model.

Second, we incorporate exogenous autonomous energy efficiency improvements to match the exogenous carbon emission profiles The autonomous energy efficiency improvement reflects the rate of change in energy intensity, i.e. the ratio of energy consumption over gross domestic product, holding energy prices constant. It is a measure of all non-price induced changes in gross energy intensity including technical developments that increase energy efficiency as well as structural changes.

Third, we recalibrate fossil fuel supply functions locally to exogenous estimates of supply elasticities. The last step assures empirical reaction of fossil fuel production to policy induced changes in world energy prices of fuels.

2. Dynamic Extension

This section provides an algebraic summary of equilibrium conditions for the intertemporal extension of the multi-region, multi-sector general equilibrium model PACE as laid out in section 1. The following key assumptions apply for the "generic" dynamic model:

- Output and factor prices are fully flexible and markets are perfectly competitive.
- Labor force productivity increases at an exogenous growth rate (Harrod-neutral technological progress).
- In equilibrium there is a period-by-period balance between exports from each region and global demand for those goods. The model adopts the Armington assumption for export and import markets of a non-energy macro good to differentiate between commodities produced for the domestic market, the export market and the import market. Fossil fuels are treated as perfect substitutes on international markets.
- In each region, a representative consumer (likewise the social planner) maximizes the present value of lifetime utility subject to (i) an intertemporal balance of payments constraint, (ii) the constraint that the output per period is either consumed (incl. intermediate demand and exports) or invested, and (iii) the equation of motion for the capital stock, i.e. capital stocks evolve through depreciation and new investment. This renders the optimal level of consumption and investment over time.
- The agents have an infinite horizon, and their expectations are forward looking and rational. To approximate an infinite horizon model with a finite horizon model we assume that the representative consumer purchases capital in the model's post-horizon period at a price which is consistent with steady-state equilibrium growth (terminal condition).

The model is formulated as a system of nonlinear inequalities using GAMS/MPSGE (Rutherford 1999) and solved using PATH (Dirkse and Ferris 1995). The inequalities correspond to the three classes of conditions associated with a general equilibrium: (i) exhaustion of product (zero-profit) conditions for constant returns to scale producers, (ii) market clearance for all goods and factors, and (iii) income balance for the representative consumers in each region. The fundamental unknowns of the system are three vectors: activity levels (production indices), non-negative prices, and consumer incomes. In equilibrium, each of these variables is linked to one inequality condition: an activity level to an exhaustion of product constraint, a commodity price to a market clearance condition, and a consumer income variable to an income definition equation. An equilibrium allocation determines production, prices and incomes.

In the following algebraic exposition, the notation Π^X is used to denote the zero-profit function of activity *X*. Formally, all production activities exhibit constant returns to scale, hence differentiating Π^X with respect to input and output prices provides compensated demand and supply coefficients, which appear subsequently in the market-clearance conditions. All prices are expressed as present values.

2.1 Exhaustion of Product Conditions

Macro Good Production

Aggregate output in region *r* describes the supply of the non-energy macro good to the domestic market and export market. A separable nested constant elasticity of substitution (CES) cost function is employed to specify the substitution possibilities between capital (*K*), labor (*L*) and an energy composite (*E*). At the top level, a constant elasticity describes the substitution possibilities between the energy aggregate and the aggregate of labor and capital. At the second level capital and labor trade off with a unitary elasticity of substitution. On the output side, production is split between goods produced for the domestic market and goods produced for the export market according to a constant elasticity of transformation. The (intra-period) zero-profit condition for the production of the macro good is:

$$\Pi_{rt}^{Y} = (\theta_r^X p_{rt}^{X^{1+\eta_r}} + (1-\theta_r^X) p_{rt}^{1+\eta_r})^{\frac{1}{1+\eta_r}} - \left[\theta_r^{EY} (\frac{p_{rt}^{EY}}{\beta_{rt}})^{1-\sigma_r^{KLE}} + (1-\theta_r^{EY}) \left(w_{rt}^{\alpha_r} v_{rt}^{1-\alpha_r} \right)^{1-\sigma_r^{KLE}} \right]^{\frac{1}{1-\sigma_r^{KLE}}} = 0$$

where:

p_{rt}^X output price of macro good produced in region *r* and period *t* for export market,

p_{rt} output price of macro good produced in region *r* and period *t* for domestic market,

p_{rt}^{EY} price of industrial energy aggregate for macro good production in region *r* and period *t*,

w_{rt} wage rate in region *r* and period *t*,

v_{rt} rental price of capital services in region *r* and period *t*,

θ_r^X benchmark share of exports in macro good production of region *r*,

θ_r^{EY} benchmark share of industrial energy aggregate in macro good production of region *r*,

α_r benchmark share of labor in value-added of macro good production in region *r*,

η_r elasticity of transformation between production for the domestic market and production for the export market of region *r,*

σ_r^{KLE} elasticity of substitution between the energy aggregate and value-added in production for region *r,*

β_{rt} exogenous energy efficiency improvement index, which measures changes in technical efficiency for region *r* in period *t*,

and

Y_{rt} associated dual variable which indicates the activity level of macro good production in region *r* and period *t.*

Fossil Fuel Production

The production of fuels requires inputs of domestic supply (macro good) and a fuel-specific factor which can be thought of as a sector-specific resource.[1] The zero-profit condition has the form:

$$\prod_{rt,ff}^{F} = p_t^{ff} - \left[\theta_r^{ff} q_{rt}^{ff^{\,1-\sigma_r^{ff}}} + (1-\theta_r^{ff}) p_{rt}^{A^{\,1-\sigma_r^{ff}}} \right]^{\frac{1}{1-\sigma_r^{ff}}} = 0 \qquad \text{ff} \in \{\text{COA,OIL,GAS}\}$$

where:

p_t^{ff} world market price of fossil fuel *ff* in period *t,*

q_{rt}^{ff} price of fuel-specific resource for production of fossil fuel *ff* in region *r* and period *t,*

p_{rt}^{A} Armington price of macro good in region *r* and period *t,*

θ_r^{ff} benchmark share of fuel-specific resource for fossil fuel production in region *r,*

σ_r^{ff} elasticity of substitution between the fuel-specific resource and non-energy inputs in fossil fuel production of region *r,*

and

$F_{rt,ff}$ associated dual variable which indicates the activity level of fossil fuel production *ff* in region *r* and period *t.*

[1] A constant returns to scale production function with convex levelsets exhibits decreasing returns to scale in *remaining* factors when one or more inputs are in fixed supply. We exploit this result in representing a decreasing returns to scale function through a constant returns to scale activity which uses the fuel-specific factor.

The value of the elasticity of substitution σ_r^{ff} between non-energy inputs and the fuel-specific resource determines the price elasticity of fossil fuel supply ε_r^{ff} at the reference point, according to the relation:

$$\varepsilon_r^{ff} = \sigma_r^{ff} \frac{\theta_r^{ff}}{1-\theta_r^{ff}} .$$

Armington Production

Inputs of the macro good into energy production, investment demand and final consumption are a composite of a domestic and imported variety which trade off with a constant elasticity of substitution. The corresponding zero profit condition for the production of the Armington good is given by:

$$\prod_{rt}^{A} = p_{rt}^{A} - \left[\theta_r^A \, p_{rt}^{1-\sigma_r^A} + (1-\theta_r^A)\left[\left(\sum_s \theta_{sr}^M \, p_{st}^{X\;1-\sigma_r^M}\right)^{\frac{1}{1-\sigma_r^M}}\right]^{1-\sigma_r^A}\right]^{\frac{1}{1-\sigma_r^A}} = 0$$

where:

θ_r^A benchmark share of domestic macro input into Armington production in region *r*,

θ_{sr}^M benchmark share of imports from region *s* (aliased with index *r*) in total macro good imports of region *r*,

σ_r^A Armington elasticity of substitution between domestic macro good and imported macro good aggregate for region *r*,

σ_r^M elasticity of substituion between macro good imports for region *r*,

and

A_{rt} associated dual variable which indicates the activity level of Armington production in region *r* and period *t.*

Production of the Industrial Energy Aggregate

Energy inputs to the macro production are a nested separable CES aggregation of oil, gas and coal. Gas and oil trade off as relatively close substitutes in the lower nest of the energy composite; at the next level the oil and gas composite combines with coal at a lower rate. The zero-profit condition for the production of the industrial energy aggregate is:

$$\prod_{rt}^{EY} = p_{rt}^{EY} - \{\theta_r^{COA}(p_t^{COA} + pcarb_{rt}CO2_{COA})^{1-\sigma_r^{COA}} + (1-\theta_r^{COA})$$

$$[\theta_r^{OIL}(p_t^{OIL} + pcarb_{rt}CO2_{OIL})^{1-\sigma_r^{LQ}} + (1-\theta_r^{OIL})(p_t^{GAS} + pcarb_{rt}CO2_{GAS})^{1-\sigma_r^{LQ}}]^{\frac{1-\sigma_r^{COA}}{1-\sigma_r^{LQ}}}\}^{\frac{1}{1-\sigma_{COA}}} = 0$$

where:

$pcarb_{rt}$ carbon price in region *r* and period *t*,

$CO2_{ff}$ physical carbon coefficient for fossil fuels,

θ_r^{COA} benchmark share of coal input into industrial energy aggregate of region *r*,

θ_r^{OIL} benchmark share of the oil input into the gas and oil composite of industrial energy production in region *r*,

σ_r^{COA} elasticity of substitution between coal and the gas and oil composite in industrial energy production of region *r*,

σ_r^{LQ} elasticity of substitution between gas and oil in industrial energy production of region *r*,

and

EY_{rt} associated dual variable which indicates the activity level of industrial energy aggregate production in region *r* and period *t*.

Production of the Household Energy Aggregate

Energy demanded by the household is a CES aggregate of fossil fuels. The zero-profit condition for the production of the household energy aggregate has the form:

$$\prod_{rt}^{EC} = p_{rt}^{EC} - \left(\sum_{ff}\theta_{r,ff}^{EC}(p_t^{ff} + pcarb_{rt}CO2_{ff})^{1-\sigma_r^{EC}}\right)^{\frac{1}{1-\sigma_r^{EC}}} = 0$$

where:

p_{rt}^{EC} price of household energy aggregate for region *r* and period *t* ,

$\theta_{r,ff}^{EC}$ benchmark share of fossil fuel input *ff* in the household energy aggregate of region *r*,

σ_r^{EC} elasticity of substitution between fossil fuel inputs within the household energy aggregate,

and

EC_{rt} associated dual variable which indicates the activity level of household energy aggregate production in region *r* and period *t*.

Production of the Household Consumption Aggregate

In final consumption demand the household energy aggregate trades off with the macro good at a constant elasticity of substitution:

$$\prod_{rt}^{C} = p_{rt}^{C} - \left(\theta_r^C \, p_{rt}^{A^{1-\sigma_r^C}} + (1-\theta_r^C) \, p_{rt}^{EC^{1-\sigma_r^C}} \right)^{\frac{1}{1-\sigma_r^C}} = 0$$

where:

p_{rt}^{C} price of household consumption aggregate for region *r* and period *t*,

θ_r^C benchmark share of macro good into aggregate household demand of region *r*,

σ_r^C elasticity of substitution between macro good and energy aggregate in household consumption demand of region *r*,

and

C_{rt} associated dual variable which indicates the activity level of household consumption in region *r* and period *t*.

Backstops for Industry and Household Energy Aggregate

For each region there is a carbon-free backstop for the industrial energy aggregate and the household aggregate. This backstop is available in infinite supply at a price which is calculated to be a multiple of the macro good price. Below, we take explicit account of the non-negativity constraint for backstop production:

$$\prod_{rt}^{\tau} = p_{rt}^{\tau} - a_r^{\tau} p_{rt}^{A} \le 0 \qquad \tau \in \{BC, BY\}$$

where:

p_{rt}^{τ} price of energy backstop for industry (τ = BY) or household (τ = BC) ,

a_r^{τ} multiplier of the macro good price index for industrial energy backstop (τ = BY) or household energy backstop (τ = BC),

and

BY_{rt} ,BC_{rt} are the associated dual variables which indicate the activity levels of backstop energy production in region *r* and period *t* for industries or households.

Capital Stock Formation and Investment

An efficient allocation of capital, i.e. investment over time assures the following intertemporal zero-profit conditions which relates the cost of a unit of investment, the return to capital and the purchase price of a unit of capital stock in period *t:* [2]

$$\Pi_{rt}^{K} = p_{rt}^{K} - v_{r}^{\tau} - (1-\delta)\, p_{r,t+1}^{K} = 0$$

and

$$\Pi_{rt}^{I} = p_{r,t+1}^{K} - p_{rt}^{I} = 0$$

where:

p_{rt}^{K} value (purchase price) of one unit of capital stock in region *r* and period *t*,

δ_r depreciation rate in region r,

p_{rt}^{I} cost of a unit of investment in period *t* which in our case equals p_{rt}^{A},

and

K_{rt} associated dual variable, which indicates the activity level of capital stock formation in region *r* and period *t,*

I_{rt} associated dual variable, which indicates the activity level of aggregate investment in region *r* and period *t* [3].

2.2 Market Clearance Conditions

Labor

The supply-demand balance for labor is:

$$\bar{L}_{rt} = Y_{rt} \frac{\partial \Pi_{rt}^{Y}}{\partial w_{rt}}$$

where:

$\bar{L}_{rt}$ exogenous endowment of time in region *r* and period *t.*[4]

Capital

The supply-demand balance for capital is:

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[2] The optimality conditions for capital stock formation and investment are directly derived from the maximization of lifetime utility by the representative household taking into account its budget constraint, the equation of motion for the capital stock and the condition that output in each period is either invested or consumed. Note that in our algebraic exposition we assume an investment lag of one period.

[3] As written, we have taken explicit account of the non-negativity constraint for investment.

[4] Time endowment grows at a constant rate *g*, which determines the long-run (steady-state) growth rate of the economy.

$$K_{rt} = Y_{rt} \frac{\partial \Pi_{rt}^{Y}}{\partial v_{rt}}$$

Fuel-Specific Resources

The supply-demand balance for fuel-specific resources is:

$$\overline{Q}_{rt}^{ff} = F_{rt,ff} \frac{\partial \Pi_{rt,ff}^{F}}{\partial q_{rt}^{ff}} \qquad ff \in \{COA, OIL, GAS\}$$

where:

$\overline{Q}_{rt}^{ff}$ exogenous endowment with fuel-specific resource *ff* for region *r* and period *t*.

Fossil Fuels

The supply-demand balance for fossil fuels is:

$$\sum_{r} F_{rt}^{ff} = \left(\sum_{r} EY_{rt} \frac{\partial \Pi_{rt}^{EY}}{\partial (p_t^{ff} + pcarb_t CO2_{ff})} + EC_{rt} \frac{\partial \Pi_{rt}^{EC}}{\partial (p_t^{ff} + pcarb_t CO2_{ff})} \right) \qquad ff \in \{COA, OIL, GAS\}$$

Macro Output for Domestic Markets

The market clearance condition for the macro good produced for the domestic market is:

$$Y_{rt} \frac{\partial \Pi_{rt}^{Y}}{\partial p_{rt}} = A_{rt} \frac{\partial \Pi_{rt}^{A}}{\partial p_{rt}}$$

Macro Output for Export Markets

The market clearance condition for the macro good produced for the export market is:

$$Y_{rt} \frac{\partial \Pi_{rt}^{Y}}{\partial p_{rt}^{X}} = \sum_{s} A_{st} \frac{\partial \Pi_{st}^{A}}{\partial p_{st}^{X}}$$

Industrial Energy Aggregate

The market clearance condition for the industrial energy aggregate is:

$$EY_{rt} + BY_{rt} = EY_{rt} \frac{\partial \Pi_{rt}^{EY}}{\partial p_{rt}^{EY}}$$

Household Energy Aggregate

The market clearance condition for the household energy aggregate is:

$$EC_{rt} + BC_{rt} = EC_{rt} \frac{\partial \Pi_{rt}^{EC}}{\partial \ p_{rt}^{EC}}$$

Armington Aggregate

The market clearance condition for Armington aggregate is:

$$A_{rt} = \ Y_{rt} \frac{\partial \Pi_{rt}^{Y}}{\partial \ \ p_{rt}^{A}} + C_{rt} \frac{\partial \Pi_{rt}^{C}}{\partial \ p_{rt}^{A}} + I_{r} \frac{\partial \Pi_{rt}^{I}}{\partial p_{rt}^{A}} + BY_{rt} \frac{\partial \Pi_{rt}^{BY}}{\partial p_{rt}^{A}} + BC_{rt} \frac{\partial \Pi_{rt}^{BC}}{\partial p_{rt}^{A}}$$

Household Consumption Aggregate

The market clearance condition for the household consumption aggregate is:

$$C_{rt} = D_{rt}$$

where:

D_{rt} uncompensated final demand which is derived from maximization of lifetime utility (see below).

2.3 Income Balance of Households

Consumers choose to allocate lifetime income across consumption in different time periods in order to maximize lifetime utility. The representative agent in each period solves:

$$\text{Max} \ \sum_{t} \left(\frac{1}{1+\rho_{r}} \right)^{t} u_{r}(C_{rt})$$

$$\text{s.t.} \ \sum_{t} p_{rt}^{C} C_{rt} \ = M_{r}$$

where:

u_r instantaneous utility function of representative agent in region *r*,

ρ_r time preference rate of representative agent in region *r*,

and

M_r lifetime income of representative agent in region *r*.

Lifetime income *M* is defined as:

$$M_r = p_{r0}^K \overline{K}_{r0} + \sum_t w_{rt} \overline{L}_{rt} + \sum_{ff} q_{rt}^{ff} \overline{Q}_{rt}^{ff}$$

$$+ \sum_t \sum_{ff} pcarb_{rt} CO2_{ff} \left(EY_{rt} \frac{\partial \Pi_{rt}^{EY}}{\partial (p_t^{ff} + CO2_{ff}\; pcarb_{rt})} + EC_{rt} \frac{\partial \Pi_{rt}^{EC}}{\partial (p_t^{ff} + CO2_{ff}\; pcarb_{rt})} \right)$$

where:

$\overline{K}_{r0}$ initial capital stock in region *r*.

With isoelastic lifetime utility the instantaneous utility function is given as:

$$u_r(C_{rt}) = \frac{C_{rt}^{1-\frac{1}{\mu_r}}}{1-\frac{1}{\mu_r}}$$

where:

μ_r constant intertemporal elasticity of substitution.

The uncompensated final demand function D_{rt} is then derived as:

$$D_{rt}(p_{rt}^C, M) = \frac{(1+\rho_r)^{-t\mu_r}}{\sum_t (1+\rho_r)^{-t^{\mu_r}} p_{rt}^{C^{1-\mu_r}}} \frac{M}{p_{rt}^{C^{\mu_r}}}$$

2.4 Terminal Constraints

The finite horizon poses some problems with respect to capital accumulation. Without any terminal constraint, the capital stock at the end of the model's horizon would have no value and this would have significant repercussions for investment rates in the periods leading up to the end of the model horizon. In order to correct for this effect we define a terminal constraint which forces terminal investment to increase in proportion to final consumption demand:[5]

$$\frac{I_{Tr}}{I_{T-1,r}} = \frac{C_{Tr}}{C_{T-1,r}}.$$

[5] This constraint imposes balanced growth in the terminal period but does not require that the model achieves steady-state growth.

2.5 Summary of Key Elasticities

Table 9 summarizes the central values for key elasticities employed for the core simulations.

Table 9: Overview of key elasticities

Type of elasticity	Description	Central Value
Armington elasticity of substitution	Degree of substitutability • Between macro imports from different regions • Between the import aggregate and the domestically produced macro good	 2 1
Armington elasticity of transformation	Degree of substitutability between macro good produced for the domestic market and macro good destined for the export market	2
Price elasticity of fossil fuel supply	Degree of response of international fossil fuel supply to changes in fossil fuel price	1 (coal), 4 (gas), 8 (oil)
Elasticity of substitution between non-energy and energy composite in production and final demand	This value increases linearly over time between a short-run value of 0.2 and the long-run value of 0.8 to reflect empirical evidence on differences between short-run and long-run adjustment costs (Lindbeck, 1983)	0.2 (short run: 2000) 0.8 (long run: 2050)
Interfuel elasticity of substitution	Degree of substitutability between fossil fuels (fuel switching)	0.5 (final demand) 2[a],1[b] (industry)

[a] between oil and gas [b] between coal and the oil-gas aggregate

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