DEMETER 1 with Carbon Capturing and Storage, a technical description

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Abstract

This report describes in detail all equations for the DEMETER 1.CCS model, for which a previous version has been used in various papers in international journals. This version is extended with carbon capturing and storage.

Keywords: energy, carbon taxes, endogenous technological change, niche markets

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1. Introduction

The DEMETER model has been used in a few papers already for an assessment of the importance of learning by doing in energy sources in abatement cost estimates (van der Zwaan *et al*., 2002), for an analysis of the costs of climate stabilization targets (Gerlagh and van der Zwaan, 2003), for a sensitivity analysis on abatement costs with respect to various economic parameters central to many integrated assessment models (Gerlagh and van der Zwaan 2004), and for an assessment of niche markets for carbon-free energy sources in abatement policies (Gerlagh *et al*. 2004). The model presented here extends the model DEMETER-1 model with a description of carbon capturing and sequestration.

2. Model basics

The DEMETER model is written in GAMS, as a set of equilibrium conditions solved using the CONOPT solver. For policy scenarios, the dynamic paths for policy variables are calculated that maximize aggregated and discounted welfare (Eq. 2) subject to instrument and climate change constraints. In mathematical terms, the program maximizes welfare subject to both instrument and climate change constraints and the equilibrium conditions. The model is truncated after 30 periods of 5 years, referring to the period 2000-2150. The investment share in 2150 is fixed, while those final-period prices for which the equations still include future variables (i.e., that involve dynamic relations) are resolved by using steady state conditions.

The model time periods are denoted by t=1,…,∞. The model distinguishes one representative consumer, three representative producers (also referred to as sectors), and a public agent that can set emission taxes to reduce carbon dioxide emissions. Producers are denoted by superscripts j=C,F,N, for the producer of the final good or consumption good, the producer of energy based on fossil-fuel technology, and the producer of energy based on carbon-free technology. There are four goods for which an equilibrium price is determined that brings supply and demand in equilibrium: the final good with price λ_t normalized to unity, λ_t=1, fossil fuel energy, with price μ_t^F, carbon-free energy with price μ_t^N, and labour with price w_t. We use β_t^τ as the price deflator for the final good from period t to period τ. So, $\beta_t^\tau = 1/[(1+r_t)(1+r_{t+1})\ldots(1+r_{\tau-1})]$, where r_t is the real interest rate. By definition, $\beta_t^t \equiv 1$ and $\beta_t^\tau = 1/\beta_\tau^t$. When convenient, we also use $\beta_t = \beta_t^{t+1} \equiv 1/(1+r_t)$. Figure 1 presents a schematic overview of the model flows. The time lag between investments and capital used as a production factor is represented through an "L" on top of the flow arrows.[2]

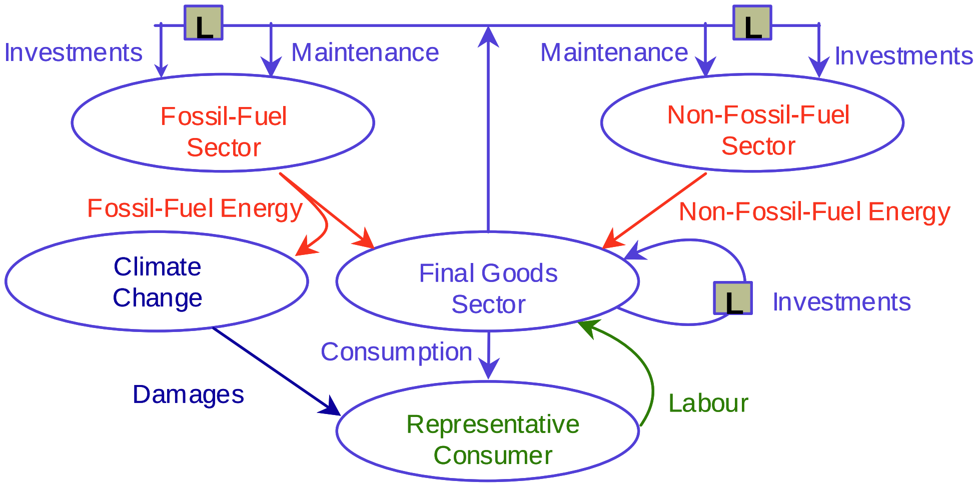

FIGURE 1. *DEMETER schematic overview of flows*

The final good is produced by sector j=C, where output is denoted by Y^C. The same good is used for consumption, investments I in all three sectors and for operating and maintenance M (as usually distinguished in energy models) in both energy sectors j=F,N. We also distinguish a separate carbon capture and storage (CCS) activity for which investments and maintenance are required.

$$C_t + I_t^C + I_t^F + I_t^{CCS} + I_t^N + M_t^F + M_t^{CCS} + M_t^N = Y_t^C . \tag{1}$$

We distinguish operating & maintenance costs, on the one hand, and investments costs, on the other hand, in line with the bottom up energy system models (cf. McDonald and Schrattenholzer, 2001). Fossil fuel energy is demanded by the final goods sector j=C and supplied by the fossil-fuel sector j=F. Carbon-free energy is demanded by the final goods sector j=C and supplied by the carbon-free energy sector j=N. Labour L_t is demanded by the final goods sector j=C and supplied inelastically by the consumers. Finally, the public agent may levy a tax τ_t on emissions Em_t produced by the final good sector when using fossil-fuel energy sources.

The representative consumer

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[2] The complete GAMS code is available through the internet, via the web-page of the first author: www.vu.nl/ivm/organisation/staff/reyer_gerlagh.html.

We assume there is one representative consumer who maximizes welfare subject to a budget constraint:

$$W = \sum_{t=1}^{\infty} (1+\rho)^{1-t} L_t \ln(C_t / L_t) , \tag{2}$$

where W is total welfare, ρ is the pure time preference, and C_t / L_t is consumption per capita. Welfare optimisation gives the Ramsey rule as a first-order-condition for consumption,

$$\beta_t = (C_t / L_t)/((1+\rho)(C_{t+1}/L_{t+1})) . \tag{3}$$

The final good producer

The representative final good producer maximizes the net present value of the cash flows:

$$\text{Max} \sum_{t=1}^{\infty} \beta_0^t (Y_t^C - I_t^C - w_t L_t - \mu_t^F Y_t^F - \mu_t^N Y_t^N) , \tag{4}$$

subject to the production constraints (5)-(11), given below. Revenues consist of output Y_t^C, expenditures consist of investments, I_t^C (one period ahead), labour L_t at wage w_t, fossil-fuel energy Y_t^F at price μ_t^F, and carbon-free energy, Y_t^N at price μ_t^N. First order conditions are given in the appendix.

To describe production, DEMETER accounts for technology that is embodied in capital installed in previous periods. It therefore distinguishes between production that uses the vintages of previous periods, and production that uses the newest vintage for which the capital stock has been installed in the directly preceding period. The input and output variables, as well as prices, associated with the most recent vintages are denoted by tildes (~). For every vintage, the production of the final good is based on a nested CES-function, using a capital-labour composite, $\widetilde{Z}_t$, and a composite measure for energy services, $\widetilde{E}_t$, as intermediates:

$$\widetilde{Y}_t^C = ((A_t^1 \widetilde{Z}_t)^{(\gamma-1)/\gamma} + (A_t^2 \widetilde{E}_t)^{(\gamma-1)/\gamma})^{\gamma/(\gamma-1)} , \qquad (\tilde{\lambda}_t^2) \tag{5}$$

where A_t^1 and A_t^2 are technology coefficients, and γ is the substitution elasticity between $\widetilde{Z}_t$ and $\widetilde{E}_t$. Notice that the Lagrange variable for the profit maximization program is given between brackets. The capital-labour composite $\widetilde{Z}_t$ is defined as:

$$\widetilde{Z}_t = (I_{t-1}^C)^{\alpha}(\widetilde{L}_t)^{1-\alpha}, \qquad (\widetilde{\theta}_t) \qquad (6)$$

which says that the capital/labour composite has fixed value share α for capital. Note that new capital is by definition equal to the investments of one period ahead, $\widetilde{K}_t^j = I_{t-1}^j$.

We model energy services $\widetilde{E}_t$ as consisting of a CES aggregate of energy produced by the sectors F and N:

$$\widetilde{E}_t = ((\widetilde{Y}_t^F)^{(\sigma-1)/\sigma} + (\widetilde{Y}_t^N)^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}, \qquad (\widetilde{\chi}_t) \qquad (7)$$

where σ is the elasticity of substitution between F and N. The CES aggregation allows for a strictly positive demand for the new technology N, if the price of the carbon-free energy exceeds the price of the fossil-fuel energy F even by an order of magnitude. By assuming the elasticity of substitution σ to have a (bounded) value larger than one, $1<\sigma<\infty$, it is ensured that the (expensive) new technology has at least a small but positive value share. In this way, the CES aggregation effectively represents an energy market and enables the economic system to take advantage of a diversified energy production, e.g. because different technologies exist, each having their own markets for which they possess a relative advantage. In DEMETER, shifts in energy sources are represented on the macro level, where gradual substitution of one technology for the other technology takes place when prices change. Though one could argue that the competition between energy sources will intensify (and thus the elasticity of substitution will increase) once the market share of carbon free technologies rises as a result of a carbon tax, we assume σ to be constant both for reasons of simplicity and for reasons of lack of empirical data. As we will argue in section 3, there is not much empirical evidence on the value of σ.

One part of production employs the new vintage, the other part employs the old capital stock that carries over from the previous period. All flows, output, use of energy, labour, and the output of emissions are differentiated between the old and the new vintages. The input/output flow in period t is equal to the corresponding flow for the new vintage, plus the corresponding flow for the old capital stock of the previous period, times a depreciation factor $(1-\delta)$.

$$Y_t^C = (1-\delta)Y_{t-1}^C + \widetilde{Y}_t^C, \qquad (\widetilde{\lambda}_t^1) \qquad (8)$$

$$Y_t^j = (1-\delta)Y_{t-1}^j + \widetilde{Y}_t^j, \qquad (\widetilde{\mu}_t^j;\ j=F,N) \qquad (9)$$

$$L_t^j = (1-\delta)L_{t-1}^j + \widetilde{L}_t \,, \qquad (\widetilde{w}_t) \qquad (10)$$

$$Em_t = (1-\delta)Em_{t-1} + \widetilde{E}m_t \,. \qquad (\widetilde{\tau}_t) \qquad (11)$$

where the last equation (11) presents the relation between total emissions Em_t and emissions of the new vintage $\widetilde{E}m_t$. Note that the equations should not be read as describing accumulation over time, and related thereto, the variables Y_t^C, Y_t^F, Y_t^N, L_t^C, Em_t, do not represent stock variables. Instead, the equations more-or-less describe the slow adjustment of production characteristics over time, as the capital stock slowly adjusts with new vintages in every period.

Energy producers

Both energy producers, the fossil fuel sector $j=F$ and the non-fossil fuel sector $j=N$ are treated almost symmetrically. The only difference is that fossil-fuel energy producers have an option to decarbonize through carbon capturing and storage. We first describe the production process abstraction from this option, that is, the production process for the non-fossil fuel sector. Production of energy, $\widetilde{Y}_t^j$ ($j=F,N$), requires investments I_{t-1}^j (in the previous period) and maintenance costs, M_t^j. For non-carbon energy producers, cash revenues consist of output value while cash payments are based on investment and maintenance costs:

$$\text{Max} \; \sum_{t=1}^{\infty} \beta_0^t (\mu_t^N Y_t^N - I_t^N - M_t^N) \,. \qquad (12)$$

Each new vintage with output $\widetilde{Y}_t^j$ requires a certain effort, measured through the variable Q, which is proportional to investments (one period ahead) and maintenance costs.

$$Q_t^j = h_t^j \tilde{Y}_t^j \,, \qquad (\varphi_{j,t};\ j=F,N) \qquad (13)$$

$$I_{t-1}^j = Q_t^j / a^j \,, \qquad (\zeta_{j,t};\ j=F,N) \qquad (14)$$

$$\tilde{M}_t^j = Q_t^j / b^j \,. \qquad (\eta_{j,t};\ j=F,N) \qquad (15)$$

where the variable h_t^j is a measure of technology variable over time, and a^j and b^j measure the constant investment and maintenance share in production costs. Technically speaking, h measures the inverse of the relative productivity compared to the long-term productivity potential for the specific process. This technology variable h is used to describe the learning process. In the long term, h_t^j converges to 1 for t converging to ∞ so that the effort Q is measured in the same units as energy output. When, in the short term, $h_t^j=2$, this means that one unit of energy output of sector j

costs twice as much effort (investments and maintenance) as compared to the situation in the far future when the learning potential has fully been appropriated.

In a similar way as expressed in the production of consumer goods (8), energy output is distinguished by vintage (9), and the same vintage approach applies to maintenance costs, M_t^j:

$$M_t^j = (1-\delta)M_{t-1}^j + \widetilde{M}_t^j . \qquad (\xi_{j,t};\ j=F,N) \qquad (16)$$

We assume that knowledge is a public good that is non-rival and non-exclusive. Thus firms will not internalize the positive spill-over effects from their investments in their prices. Hence, the productivity parameter h_t^j is treated as exogenous by the firms, and the individual firms are confronted with constant returns to scale. Profit maximization of (12) subject to (9), (13), (14), (15), and (16) gives zero profits. First order conditions are listed in the appendix.

Energy production based on fossil fuels can be confronted with a carbon tax levied on carbon dioxide emissions, and producers can choose to decarbonize energy through carbon capturing and sequestration (CCS). Energy related carbon dioxide emissions, $EnEm_t$, are proportional to the carbon content of fossil fuels, denoted by ε_t^F, but part of energy related emissions, *CCSR*, is captured through a carbon capturing and storage activity (17). Equation (18) calculates the total flow of carbon capture and storage, CCS_t, of both old and new vintages. In every period, the stock of stored carbon, S_t^{CCS}, increases with the flow of CCS_t, while a fixed share δ^{CCS} of the stored stock of carbon leaks into the atmosphere (19), and this leakage adds to future emissions (26).

$$EnEm_t = (1-\delta)EnEm_{t-1} + \varepsilon_t^F(1-CCSR_t)\tilde{Y}_t^F . \qquad (\tilde{\tau}_t) \qquad (17)$$

$$CCS_t = (1-\delta)CCS_{t-1} + CCSR_t\varepsilon_t^F\tilde{Y}_t^F . \qquad (18)$$

$$S_{t+1}^{CCS} = (1-\delta^{CCS})S_t^{CCS} + CCS_t , \qquad (19)$$

The variable *CCSR* can be understood as the carbon capturing and sequestration ratio. When convenient, we use the acronym *CCS* for the carbon capturing and storage activity, measured in metric tons of carbon, and *CCSR* for the ratio of emissions prevented through this activity. The tildes on top of the variable denote that emission intensities are vintage specific. Alternatively, we can interpret the *CCSR* variable in a broader perspective as a broad decarbonization measure, where ε_t^F is the carbon intensity of a benchmark fuel mix that is optimal without carbon tax, and *CCSR* includes all activities that reduce carbon dioxide emissions, including fuel-switching options.

Given the carbon tax and the *CCS* option, the cash flows equation (12) is adjusted to account for additional costs of investments and maintenance for *CCS*, and for the carbon tax levied on emissions:

$$\text{Max} \sum_{t=1}^{\infty} \beta_0^t (\mu_t^F Y_t^F - I_t^F - M_t^F - I_t^{CSS} - M_t^{CCS} - \tau_t \, Em_t^F) . \tag{20}$$

Similar to the production of energy described above, the carbon capturing and sequestration process is described through an effort variable Q_t^{CCS}, which is assumed a second order polynomial function of the share of carbon that is captured and sequestered:

$$Q_t^{CCS} = h_t^{CCS} (CCSR_t + \tfrac{1}{2} \kappa \, CCSR_t^2) \varepsilon \widetilde{Y}_t^F , \qquad (\varphi_{j,t};\ j{=}CCS) \tag{21}$$

Investments and maintenance costs are described through the same equations as for the production process: (14), (15), and (16). The quadratic cost curve implies that the amount of carbon that is captured and not emitted is linear in the carbon tax.

Technological change

The DEMETER model incorporates various insights from the bottom-up literature that stress the importance of internalizing learning-by-doing effects in climate change analyses. Energy production costs decrease as the experience increases through the installation of new energy vintages. In this version of DEMETER, the endogenous modeling of learning by doing is limited to the energy sectors; we have not included learning effects for overall productivity and energy efficiency. Thus, A_t^1 and A_t^2 as employed in (5) are exogenously determined by a benchmark (business as usual) growth path.

Stated in other terms, the variable h_t^j measures the costs of one unit of output $\widetilde{Y}_t^j$ as compared to potential long-term costs. For example, h_t^j=2 means that one unit of energy output of sector *j* costs twice as much investments and maintenance costs as compared to the situation in the far future when the learning effect has reached its maximum value.

To capture the process of gaining experience and a decreasing value of h_t^j, we introduce the variable X_t that represents experience; it counts accumulated installed new capacity (vintage) at the beginning of period *t*. For energy production, the new capacity is equal to the output of the new vintage. For carbon capturing and sequestration, the new capacity is the amount of emissions prevented.

$$X_{t+1}^{j} = X_{t}^{j} + \tilde{Y}_{t}^{j}\,. \qquad (j=F,N) \qquad (22)$$

$$X_{t+1}^{CCS} = X_{t}^{CCS} + CCSR_{t}\varepsilon_{t}\tilde{Y}_{t}^{F}\,. \qquad (23)$$

Furthermore, we use a scaling function $g^j(X)\rightarrow[1,\infty)$ that returns the value for h_t^j as dependent on cumulative experience at the beginning of the period, X_t^j.

$$h_{t}^{j} = g^{j}(X_{t}^{j}), \qquad (j=F,N,CCS) \qquad (24)$$

We assume $g'(.)\leq 0$, that is, production costs decrease as experience increases, and we assume $g^j(\infty)=1$, that is, production costs converge to a strictly positive floor price (minimum amount of input associated with maximum learning effect) given by the levels of a_{∞}^{j} and b_{∞}^{j}. Finally, we assume a constant learning rate $lr>0$ for technologies at the beginning of the learning curve (that is, for small values of X). This means that, initially, production costs decrease by a factor $(1-lr)$, for every doubling of installed capacity. Such decreases have been observed empirically for a large range of different technologies (IEA/OECD, 2000).

A function $g^j(.)$ that supports all these assumptions is given by:

$$g^{j}(X) = c^{j}(1-d^{j})X^{-d^{j}} + 1\,. \qquad (25)$$

where we omitted subscripts t and superscript j for the variable X, and $0<d^j<1$ measures the speed of learning, and c^j measures the size of the learning costs relative to the long-term production costs.[3] Finally, we notice that, in a model without learning-by-doing, we would have $g^j(.)=1$.

Climate change and instruments

Emissions are included in the equilibrium through equations (11) and (17). Environmental dynamics are included through the climate change dynamics from DICE99 (Nordhaus and Boyer 2000), and describe a multi-stratum system, including an atmosphere, an upper-ocean stratum, and a lower-ocean stratum. We recalibrated the DICE99 climate module parameters to fit our five-year periods, whereas DICE99 uses periods of 10 years,

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[3] The learning rate lr and the parameter d used in (25) are approximately related by the equation $d = -\ln(1-lr)/\ln 2$. For small learning rates lr, we make the approximation $d=lr/\ln 2$.

$$ATM_{t+1} = (Em_t + \overline{\overline{Em}}_t + \delta^{CCS} S_{t-1}^{ccs}) + TR_{atm}^{atm} ATM_t + TR_{ul}^{atm} UL_t \,, \tag{26}$$
$$UL_{t+1} = TR_{atm}^{ul} ATM_t + TR_{ul}^{ul} UL_t + TR_{ll}^{ul} LL_t \,, \tag{27}$$
$$LL_{t+1} = TR_{ul}^{ll} UL_t + TR_{ll}^{ll} LL_t \,, \tag{28}$$
$$F_t = 4.1^2 \ln(ATM_t \,/\, ATM_0) + EXOFORC_t \,, \tag{29}$$
$$TEMP_{t+1} = TEMP_t + \delta^T (F_t / 4.1)(\overline{T} - TEMP_t) - TR_{TEMP}^{TLOW} (TEMP_t - TLOW_t) \,, \tag{30}$$
$$TLOW_{t+1} = TLOW_t + CA_{TLOW}^{TEMP} TR_{TEMP}^{TLOW} (TEMP_t - TLOW_t) \,, \tag{31}$$

where ATM_t is the atmospheric CO_2 content, UL_t is the CO_2 content of the upper ocean layer, LL_t is the CO_2 content of the lower ocean layer, F_t is the radiative forcing, $TEMP_t$ is the atmospheric temperature increase relative to pre-industrial, and $TLOW_t$ is the ocean temperature increase. The exogenous variables are $\overline{Em}_t$ for the exogenous path of non-energy related CO_2 emissions, and $EXOFORC_t$ is the forcing caused by non-CO_2 greenhouse gases. The parameters are TR_{atm}^{ul} = 0.2128 for the per-period CO_2 transport share from the atmosphere to the upper ocean layer; TR_{ul}^{atm}=0.1760 is the per-period CO_2 transport share from the upper layer to the atmosphere, TR_{ul}^{ll}=0.0625 is the per-period CO_2 transport share from the upper layer to the lower layer, TR_{ll}^{ul}=0.0023 is the per-period CO_2 transport share from the lower layer to the upper layer, TR_{atm}^{atm} =1–TR_{atm}^{ul} is the CO_2 share remaining in the atmosphere, TR_{ul}^{ul} =1–TR_{ul}^{atm}–TR_{ul}^{ll} is the CO_2 share remaining in the upper layer, and TR_{ll}^{ll} =1–TR_{ll}^{ul} is the CO_2 share remaining in the lower layer. Finally, δ^T=0.120 is the temperature adjustment rate due to the atmospheric warmth capacity, $\overline{T}$ is the long-term equilibrium temperature change associated with a doubling of atmospheric CO_2 concentrations, TR_{TEMP}^{TLOW}=0.051 is the relative heat transport from the atmosphere to the ocean, and CA_{TEMP}^{TLOW}=0.201 is the relative warmth capacity of the atmosphere relative to the ocean.

3. Calibration and data for numerical analysis

Recall major calibration values and then make note that says: See earlier publications for calibration of all but CCS.

The extent to which CCS technologies may contribute to greenhouse gas emission control and atmospheric CO_2 concentration stabilization goals will be determined by its costs. The major cost components of CCS are related to the capturing of carbon, that is the separation and compression, its transport, and its storage, the latter including measurement, monitoring and verification. The cost of employing a full CCS system for electricity generation from a fossil-fired power plant is dominated by the cost of capture. Since so far, limited commercial experience in configuring the various components into an integrated CCS system has been obtained; the cost ranges quoted in the literature are large. These costs depend on the specific characteristics of the capture technology used and the power plant. In this paper, it is assumed that a range of CCS options is available, from a low-cost end to a high-cost end. The share of fossil fuel energy from which carbon is captured and sequestered is assumed linear in the carbon tax. The low-cost options are first used, and more expensive options are added when the carbon tax increases.

In the first period, we assume that some CCS is economic feasible at costs of around 10 \$/tC (avoided, that is, 3 \$/tCO_2 avoided). This figure is relatively low as we assume that in some cases, CCS can increase the produce of oil fields. At the high-cost end, it is assumed that if one nears the point of applying CCS to the use of all fossil fuel electricity generation, about one third of total energy demand in primary energy equivalents, costs will be as high as 150 \$/tC. This high-cost value corresponds well to the average of the typical cost ranges as currently provided by specialists in the field of CCS.[4] We note that these values imply that the application of a full-cost CCS system would typically add some 2-5 cent/kWh to the costs of electricity from a pulverised coal power plant, which is the order of magnitude of the electricity generation costs themselves.

The capture technology part in CCS systems resembles the technologies used for sulphur and nitrous oxides removal from flue gases. Flue gas desulphurisation (FGD) is a common method applied in large industrial installations like fossil power plants, used to remove sulphur oxides from the emitted exhaust gases. Power plant FGD processes are employed on large commercial scales in most industrialised countries. Selective catalytic reduction (SCR) is a widely applied industrial process for the reduction of nitrogen oxides in exhaust gases from large stationary fossil-fuel combustion installations. Worldwide, the costs of applying these technologies have decreased considerably over the past decades (Rubin *et al*., 2004a and 2004b) and learning rates for capital costs of 11% and 12% were found. We assume that CCS will follow the same route of technological progress, and we take a learning rate of 10%.

As DEMETER does not distinguish between the capture and storage parts of CCS technologies, it is supposed that this 10% learning rate is applicable to the employment of CCS at large. Still, application of the learning rate requires an estimation of the initial level of cumulative experience. No large-scale power plant has so far been retrofitted with carbon dioxide capture technology. On the other hand, carbon dioxide storage has been taking place for already a number of years at e.g. the Sleipner project (0.2-0.3 MtC/yr), in the Weyburn project (1-2 MtC/yr) and in

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[4] The IPCC (Intergovernmental Panel on Climate Change, Working Group III), in an envisaged Special Report on Carbon Dioxide Capture and Storage, is currently in the process of assembling a comprehensive overview of CCS technologies, including an assessment of their prospective costs.

West Texas (5-10 MtC/yr). For our calculations with DEMETER, we assume that the CCS cost estimates stated above are applicable when experience has cumulated to about 20 MtC/yr of CCS capacity installed.

4. Further model conditions

First order conditions for the final good producer

Maximizing net profits (4), subject to the constraints (5)-(11) yields the following first order conditions for Y_t^C, Y_t^j (*j*=*F*,*N*), L_t, , Em_t, $\tilde{Z}_t$, I_t^C, $\tilde{L}_t$, $\tilde{E}_t$, $\tilde{Y}_t^j$:

$\tilde{\lambda}_t = (1-\delta)\beta_t\tilde{\lambda}_{t+1} + 1$,	(Y_t^C),	(32)
$\tilde{\mu}_t^j = (1-\delta)\beta_t\tilde{\mu}_{t+1}^j + \mu_t^j$.	(Y_t^j, *j*=*F*,*N*),	(33)
$\tilde{w}_t^j = (1-\delta)\beta_t\tilde{w}_{t+1}^j + w_t^j$.	(L_t),	(34)
$\tau_t = \tilde{\tau}_t - (1-\delta)\beta_t\tilde{\tau}_{t+1}$.	(Em_t),	(35)
$\tilde{\theta}_t = \tilde{\lambda}_t^2(A_t^1)^{(\gamma-1)/\gamma}(\tilde{Z}_t / \tilde{Y}_t^C)^{-1/\gamma}$	($\tilde{Z}_t$),	(36)
$1 = \beta_t\tilde{\theta}_{t+1}\alpha\tilde{Z}_{t+1} / I_t^C$	(I_t^C),	(37)
$\tilde{w}_t\tilde{L}_t = (1-\alpha)\tilde{\theta}_t\tilde{Z}_{t+1}$	($\tilde{L}_t$),	(38)
$\tilde{\chi}_t = \tilde{\lambda}_t^2(A_t^2)^{(\gamma-1)/\gamma}(\tilde{E}_t / \tilde{Y}_t^C)^{-1/\gamma}$	($\tilde{E}_t$),	(39)
$\tilde{\mu}_t^j = \tilde{\chi}_t(\tilde{Y}_t^j / \tilde{E}_t)^{-1/\sigma}$.	($\tilde{Y}_t^j$; *j*=,*N*,*F*)	(40)

where the variables associated with the first order conditions are given between brackets, $\tilde{\lambda}_t$ is the shadow price for $\tilde{Y}_t^C$, that is the Lagrange variable for (8) which is the same as the Lagrange variable for (5), $\tilde{\mu}_t^j$ is the shadow price for $\tilde{Y}_t^j$, and the Lagrange variable for (9), $\tilde{w}_t$ is the shadow price for $\tilde{L}_t$, and the Lagrange variable for (10), $\tilde{\theta}_t$ is the shadow price for the labour/capital composite $\tilde{Z}_t$ and the Lagrange variable for (6), $\tilde{\chi}_t$ is the shadow price for the energy composite $\tilde{E}_t$ and the Lagrange variable for (7).

Energy producers

The non-carbon energy producers maximize net profits (12) subject to (9), (13), (14), (15), and (16). Calculating the first order conditions for Y_t^j, $\tilde{Y}_t^j$, Q_t^j, $\tilde{M}_t^j$, I_{t-1}^j, and M_t^j, we find (33) and

$\tilde{\mu}_t^N = h_t^N\varphi_t^N$,	($\tilde{Y}_t^N$)	(41)
$\varphi_t^j = \zeta_t^j + \eta_t^j$,	(Q_t^j, j=F,N,CCS)	(42)
$\tilde{\xi}_t^j = b^j\eta_t^j$,	($\tilde{M}_t^j$, j=F,N,CCS)	(43)

$$1 = a^j \beta_t \zeta_{t+1}^j \ , \qquad (I_{t-1}^j, j{=}F,N,CCS) \qquad (44)$$

$$\tilde{\xi}_t^j = (1-\delta)\beta_t \tilde{\xi}_{t+1}^j + 1 \ , \qquad (M_t^j, j{=}F,N,CCS) \qquad (45)$$

where $\tilde{\mu}_t^j$ is the shadow price for $\tilde{Y}_t^j$, and the Lagrange variable for (9), φ_t^j is the shadow price of Q_t^j and the Lagrange variable of (13), ζ_t^j and η_t^j are the Lagrange variables of (14), and (15), and $\tilde{\xi}_t^j$ is the shadow price of $\tilde{M}_t^j$.

The fossil fuel energy producers maximize net profits (20) subject to (9), (11), (13), (14), (15), (16), (17), and (21). Calculating the first order conditions for Y_t^j, Q_t^j, $\tilde{M}_t^j$, I_{t-1}^j, M_t^j, $\tilde{Y}_t^j$, and $CCSR_t$, we find (33) for *j=F*, (42), (43), (44), and (45) for *j=F,CCS*, and

$$\tilde{\mu}_t^F = h_t^F \varphi_t^F + (1 - CCSR_t)\tilde{\tau}_t \varepsilon_t^F + h_t^{CCS} \varphi_t^{CSS} \varepsilon_t^F (CCSR_t + \tfrac{1}{2}\kappa CCSR_t^2) \, , \, (\tilde{Y}_t^F) \qquad (46)$$

$$(1 + \kappa \, CCSR_t)\varphi_t^{CCS} h_t^{CCS} \geq \tau_t \perp CCSR_t \geq 0, \qquad (CCSR_t) \qquad (47)$$

respectively, where the Lagrange variable of (16), $\tilde{\tau}_t$ is the shadow price for $\tilde{E}m_t$ and the Lagrange variable for (11), which has the same value as the Lagrange variable for (17).

5. Defining instruments

We define five instruments for cutting carbon dioxide emissions: a carbon tax, a fossil fuel tax, a subsidy on non-fossil energy, a portfolio standard for the carbon emission intensity, and a portfolio standard for the non-fossil energy share. The carbon tax is included in the equations (46) and (47). The fossil fuel tax and non-carbon energy subsidy are defined as distortions between the price the energy producer receives and the final good producer pays for its energy use. Thus, equation (40) is adjusted to

$$\tilde{\mu}_t^j + tax_t^j - sub_t^j = \tilde{\chi}_t (\tilde{Y}_t^j / \tilde{E}_t)^{-1/\sigma} \ . \qquad (\tilde{Y}_t^j ; j{=},N,F) \qquad (48)$$

For the fossil fuel tax, we take $tax^F>0$, and for the subsidy on non-carbon energy, we take $sub^N>0$. There is no scenario with non-carbon energy taxes ($tax^N>0$ does not occur) or a fossil fuel subsidy ($sub^F>0$ does not occur). Both portfolio scenarios assume zero tax revenues. That is, the carbon or fossil fuel tax revenues equal the subsidy expenditures:

$$\tilde{\tau}_t^j Em_t + tax_t^F \tilde{Y}_t^F = sub_t^N \tilde{Y}_t^F \ . \qquad (49)$$

The next table summarizes the five instruments. In all scenarios, typically the time path for the instrument level is chosen endogenously to satisfy a certain objective, e.g. to maximize intertemporal (discounted and aggregated) welfare subject to a carbon dioxide stabilization constraint.

TABLE 1. *Definitions of emission reduction instruments*

Carbon tax	$\tau_t \geq 0$	$tax_t^F = 0$	$sub_t^N = 0$	(49) not binding
Fossil fuel tax	$\tau_t = 0$	$tax_t^F \geq 0$	$sub_t^N = 0$	(49) not binding
Non-carbon subsidy	$\tau_t = 0$	$tax_t^F = 0$	$sub_t^N \geq 0$	(49) not binding
Portfolio standard for carbon emission intensity	$\tau_t \geq 0$	$tax_t^F = 0$	$sub_t^N \geq 0$	(49) binding
Portfolio standard for non-fossil energy share	$\tau_t = 0$	$tax_t^F \geq 0$	$sub_t^N \geq 0$	(49) binding

One of the main differences between the carbon tax and the fossil fuel tax is that the in the latter case energy producers have no incentive to apply CCS (47). Only when carbon emissions are taxed it becomes interesting to invest in CCS technologies, so that only in the carbon tax and carbon emission standards scenarios, CCS implementation materializes. In the other scenarios there is no incentive to complement power plants (or other carbon-emitting energy uses) with costly CCS technologies. The fossil fuel tax and non-fossil fuel portfolio standard scenarios may be applied when e.g. the differentiation in taxes between different fuels is too difficult or becomes too tedious. In both portfolio standard scenarios, there is no net tax on energy.

6. Measuring welfare changes

We apply three measures for welfare changes, insofar as welfare depends on the consumption of the final good. That is, we do not go into welfare calculations associated with climate change. The first measure of welfare change is based on the net present value of the change in the consumption bundle, evaluated at benchmark prices,

$$\Delta W = \sum_{t=1}^{\infty} \overline{\beta}_1^t (C_t - \overline{C}_t), \tag{50}$$

where a bar on top of the variable implies that its value is taken from the benchmark equilibrium path.

The second measure of welfare change uses the principle of equivalent variation. Recall that the vector β_1 denotes the price depreciation over time, that is, it describes for all periods t the relative prices from period t to period 1 for the consumer good. Now, let $\bar{\beta}$ be the price vector for a benchmark scenario, and β the vector for the alternative scenario. Similarly, let $\bar{W}$ and W be the successive welfare levels. Finally, let $E(W,\beta)$ be the expenditure function, expressed in consumer goods priced at the first period. Thus, $E(W,\beta)$ is the minimum net present value of expenditures to achieve the welfare level W when prices depreciate by β. When W and β are from the same equilibrium path, it follows from utility maximization that

$$E(W,\beta) = \sum_{t=1}^{\infty} \beta_1^t C_t \ , \tag{51}$$

where we recall that $\beta_1^1 = 1$. Notice however that this equation does not hold when W and β are from different paths. The equivalent variation is defined as

$$EV = E(W,\bar{\beta}) - E(\bar{W},\bar{\beta}). \tag{52}$$

When writing out equation (52) by use of (51), it may look as if the equivalent variation measure is equal to the net present value of changes in consumption (50). This is not so. The reason is that equation (51) does not hold for the first expenditure function in (52), since W and β are from different paths. Nonetheless, as long as β does not change too much between the benchmark and the alternative scenario, the two measures (50) and (52) will be very close. In such cases, the two measures can be used as a check to make sure that no errors occurred in the GAMS welfare calculation code.

Whereas (52) presents the definition of equivalent variation, we will now work out how it is calculated. Consider an alternative path denoted by subscript 1, and with welfare level, W_1, with difference A compared to the benchmark, $W_1=\bar{W}+A$. Because of constant cost-shares for consumption in each period, given by the constants $(1+\rho)^{-t}L_t$ in front of the log in the welfare function (2), we will have that the consumption stream supporting the expenditure function based on benchmark prices $\bar{\beta}$ for any two welfare levels in (51) have a constant ratio, with value $\exp(A/\kappa)$, where

$$\kappa = \sum_{t=1}^{\infty} (1+\rho)^{1-t} L_t \ . \tag{53}$$

The aggregated and discounted expenditure thus also differ a factor $\exp(A/\kappa)$:

$$E(W_1,\overline{\beta}) = \exp(A/\kappa)\ E(W_0,\overline{\beta}), \tag{54}$$

and we have for the equivalent variation

$$EV = (\exp((W-\overline{W})/\kappa)-1)\ E(\overline{W},\overline{\beta}). \tag{55}$$

We also present a third measure of welfare changes that can be useful for the analysis. This measure is more complex as it requires the full path information when moving from one scenario to another scenario. It is based on an integral over market distortions, evaluating the market distortions along the scenario adjustment path. Recall that we consider welfare as only dependent on consumption without paying attention to the climate change feed back. In this interpretation of welfare, the model has three distortions due to which costs and benefits of one unit of output do not match. First, the carbon tax distorts the emission market. Its cumulative welfare costs are given by

$$\Delta W_{carbon\ tax} = \sum_{t=1}^{\infty} \int_{E_{BAU,t}}^{E_{S,t}} \beta_1^t \tau_t \,\mathrm{d}E \ , \tag{56}$$

where E_{BAU} is path of benchmark emission levels, E_S are the emission levels for the specific scenario, and τ is the carbon tax level. The second market distortion is cased by the learning externality for both fossil fuels and non-fossil fuels

$$\Delta W^j_{learning} = \sum_{t=1}^{\infty} \int_{\tilde{Y}^j_{BAU,t}}^{\tilde{Y}^j_{S,t}} \beta_1^{t+1} \pi^j_{t+1} \,\mathrm{d}\tilde{Y}^j_t \ , \tag{57}$$

where π^j_t is the learning spillover value of one unit of new capacity. This variable π^j_t specifies the decrease in prices that would internalize the learning spillover; it measures the wedge between the average and marginal costs of one unit of capacity, where the average costs are assumed to be the market price. Thus, π measures the shadow price of equation (22). Its value is found by taking the derivative for X^j_t

$$\pi^j_t = \beta_t \pi^j_{t+1} + g^j{}'(.)\varphi^j_t \tilde{Y}^j_t \ . \qquad (X^j_t), \tag{58}$$

The third market distortion is linked to the learning externality for CCS. Its value is given by

$$\Delta W_{learning\ CCS} = \sum_{t=1}^{\infty} \int_{0}^{CCS_{S,t}} \beta_1^t \pi_{t+1}^{CCS} \,\mathrm{d}CCS_t \quad , \tag{59}$$

The marginal social value of one unit of CCS not captured in its market price, is given by

$$\pi_t^{CCS} = \beta_t \pi_{t+1}^{CCS} + g^{CCS}\,'(.)\varphi_t^{CCS} Q_t^{CCS} / h_t^{CCS} \ . \tag{60}$$

Now, the change in welfare when moving from BAU to an alternative scenario *S*, is given by the sum of the three market distortions.

$$\Delta W = \Delta W_{carbon\ tax}. + \Delta W^F_{learning} + \Delta W^N_{learning}. + \Delta W_{learning\ CCS}. \tag{61}$$

Since the carbon tax and learning welfare change measures are based on an integral, we have to use a local approximation of this integral. For the shadow price τ and π, we take the average between the values found for the BAU and the alternative scenario, and we multiply this shadow price by the change in emissions, change in capacities, and change in CCS, respectively. When the BAU and alternative scenario differ too much (e.g. for a stringent climate stabilization scenario), we divide the scenario adjustment in various small steps.

As a check, we compare the three measures of welfare changes (50), (52), and (61), which should be approximately equal. The third measure allows us to interpret welfare changes in terms of market distortions. When comparing BAU with a stabilization scenario, we can calculate which part of the costs is due to a carbon tax burden, and how much the costs are reduced by gained learning insofar this is in excess of the learning costs for non-fossil fuels and for CCS. It also gives us the increase in costs (decrease in welfare) due to lower learning levels for fossil fuels.

7. Full specification of equilibrium equations and variables

We can now characterize the equilibrium by its variables, its equations and its first order conditions. The endogenous variables are C_t, Y_t^C, $\widetilde{Y}_t^C$, Y_t^j (j=F,N), $\widetilde{Y}_t^j$ (j=F,N), L_t, $\widetilde{L}_t$, Em_t, $\widetilde{E}m_t$, $\widetilde{Z}_t$, I_t^C, I_{t-1}^j (j=F,N), $\widetilde{E}_t$, Q_t^j, M_t^j, $\widetilde{M}_t^j$, and h_t^j for endogenous technology. Prices are μ_t^j for the two energy sources, and w_t for labour. The price deflator is β_t, and shadow prices are $\widetilde{\lambda}_t$ for $\widetilde{Y}_t^C$, $\widetilde{\mu}_t^j$ for $\widetilde{Y}_t^j$, $\widetilde{w}_t$ for $\widetilde{L}_t$, $\widetilde{\tau}_t$ for $\widetilde{E}m_t$, $\widetilde{\theta}_t$ for $\widetilde{Z}_t$, $\widetilde{\chi}_t$ for $\widetilde{E}_t$, φ_t^j for Q_t^j, $\widetilde{\xi}_t^j$ for $\widetilde{M}_t^j$ and ζ_t^j and η_t^j are the Lagrange variables of (14), and (15). The final good commodity balance is given by (1). The welfare level is defined by the welfare function (2) and consumer behavior by the first order condition (3). Production of the final good is defined by production identities (5)-(11), and the first order conditions (32)-(40). Energy production is defined by production identities (16), (13), (14), and (15), and the first order conditions (46)-(45). Endogenous technological change is defined by (22) and (24). Finally, climate change is defined by (26) and (30).

As we have a vintage model, the flows one period before the first period, that is in period t=0, determine the flows of the old vintage in period t=1, and are exogenous to the model: $Y_0^C, I_0^C, I_0^F, I_0^N, M_0^F, M_0^N, X_1^F$ and X_1^N have to be specified as input (or initiation) parameters at the start of the model simulation, as they result from investment decisions before period t=1.

For computational efficiency, we leave various variables out of the equilibrium, and calculate these variable in advance (ex-ante), or afterwards (ex-post). For example, labour is supplied inelastically, and is assumed to increase proportionally with population levels. The labour flow available for each new vintage $\widetilde{L}_t$ can, ex-ante, be calculated by (10). Consequently, we can leave the first order conditions for L_t and $\widetilde{L}_t$, (34) and (38), out of the model, and calculate w_t^j and $\widetilde{w}_t^j$ afterwards, when required.

First order conditions (46), and (41) are rewritten as zero profit conditions:

$$\tilde{\mu}_t^F \tilde{Y}_t^F = \varphi_t^F Q_t^F + \varphi_t^{CSS} Q_t^{CCS} + \tau_t\, Em_t \ , \qquad (\tilde{Y}_t^F) \tag{62}$$
$$\tilde{\mu}_t^N \tilde{Y}_t^N = \varphi_t^N Q_t^N \ , \qquad (\tilde{Y}_t^N) \tag{63}$$

respectively.

Terminal conditions

The model source code is written in GAMS, and solved using the CONOPT solver. It is truncated after T periods. For the last period T, welfare is given an extra weight for the omitted tail. For this purpose, we add the dummy Ω_t^W, with $\Omega_t^W=1$ for $t<T$, and $\Omega_T^W=1/\rho$. Equation (2) now becomes (64). Similarly, all dynamic price equations (26), (29), (39) are adjusted with a dummy Ω_t^λ, with $\Omega_t^\lambda=1$ for $t<T$, and $\Omega_t^\lambda=1/(1-\beta(1-\delta))$, to set in the final period the vintage prices λ, τ, and ξ at their steady state value

Integration

Some variables, such as Y_t^j and its price μ_t^j (j=F,N) are calculated afterwards omitting equation (9) and (33) out of the equilibrium equation set. This brings us to the full list of equilibrium equations:

Welfare:

$$W = \sum_{t=1}^{T} (1+\rho)^{1-t}\, \Omega_t^W\, Pop_t \ln(C_t / Pop_t)\,, \tag{64}$$

Production and consumption:

$$C_t + I_t^C + I_t^F + I_t^{CCS} + I_t^N + M_t^F + M_t^{CCS} + M_t^N = Y_t^C\,. \tag{1}$$

$$Y_t^C = (1-\delta)Y_{t-1}^C + \widetilde{Y}_t^C\,, \qquad (\tilde{\lambda}_t^1) \tag{8}$$

$$\widetilde{Y}_t^C = ((A_t^1 \widetilde{Z}_t)^{(\gamma-1)/\gamma} + (A_t^2 \widetilde{E}_t)^{(\gamma-1)/\gamma})^{\gamma/(\gamma-1)}\,, \qquad (\tilde{\lambda}_t^2) \tag{5}$$

$$\widetilde{Z}_t = (I_{t-1}^C)^{\alpha} (\widetilde{L}_t)^{1-\alpha}\,, \qquad (\widetilde{\theta}_t) \tag{6}$$

$$\widetilde{E}_t = ((\widetilde{Y}_t^F)^{(\sigma-1)/\sigma} + (\widetilde{Y}_t^N)^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}\,, \qquad (\widetilde{\chi}_t) \tag{7}$$

$$X_{t+1}^j = X_t^j + \widetilde{Y}_t^j\,. \qquad (j{=}F,N) \tag{22}$$

$$X_{t+1}^{CCS} = X_t^{CCS} + CCSR_t \varepsilon_t \tilde{Y}_t^F\,. \tag{23}$$

$$h_t^j = g^j(X_t^j), \qquad (j{=}F,N,CCS) \tag{24}$$

$$Q_t^j = h_t^j \tilde{Y}_t^j\,, \qquad (\varphi_{j,t};\ j{=}F,N) \tag{13}$$

$$Q_t^{CCS} = h_t^{CCS}(CCSR_t + \tfrac{1}{2}\kappa\, CCSR_t^2)\varepsilon \widetilde{Y}_t^F\,, \qquad (\varphi_{j,t};\ j{=}CCS) \tag{21}$$

$$Q_t^j = a^j I_{t-1}^j\,, \qquad (\zeta_{j,t};\ j{=}F,N,CCS) \tag{14}$$

$$Q_t^j = b^j \widetilde{M}_t^j\,. \qquad (\eta_{j,t};\ j{=}F,N) \tag{15}$$

$$M_t^j = (1-\delta)M_{t-1}^j + \widetilde{M}_t^j\,. \qquad (\xi_{j,t};\ j{=}F,N) \tag{16}$$

First order conditions:

$$\beta_t = (C_t / L_t)/((1+\rho)(C_{t+1}/L_{t+1}))\,. \tag{3}$$

$$\tilde{\lambda}_t = (1-\delta)\beta_t \tilde{\lambda}_{t+1} + \Omega_t^{\lambda}\,, \qquad (Y_t^C), \tag{65}$$

$$\tau_t = \Omega_t^{\lambda} \tilde{\tau}_t - (1-\delta)\beta_t \tilde{\tau}_{t+1}\,. \qquad (Em_t), \tag{66}$$

$$\widetilde{\theta}_t = \widetilde{\lambda}_t^2 (A_t^1)^{(\gamma-1)/\gamma} (\widetilde{Z}_t / \widetilde{Y}_t^C)^{-1/\gamma} \qquad (\widetilde{Z}_t), \tag{36}$$

$$1 = \beta_t \widetilde{\theta}_{t+1} \alpha \widetilde{Z}_{t+1} / I_t^C \qquad (I_t^C), \tag{37}$$

$$\widetilde{\chi}_t = \widetilde{\lambda}_t^2 (A_t^2)^{(\gamma-1)/\gamma} (\widetilde{E}_t / \widetilde{Y}_t^C)^{-1/\gamma} \qquad (\widetilde{E}_t), \tag{39}$$

$$\tilde{\mu}_t^j + tax_t^j - sub_t^j = \tilde{\chi}_t (\tilde{Y}_t^j / \tilde{E}_t)^{-1/\sigma}\,. \qquad (\widetilde{Y}_t^j;\ j{=},N,F) \tag{48}$$

$$\tilde{\mu}_t^F \tilde{Y}_t^F = \varphi_t^F Q_t^F + \varphi_t^{CSS} Q_t^{CCS} + \tau_t\, Em_t\,, \qquad (\tilde{Y}_t^F) \tag{62}$$

$$\tilde{\mu}_t^N \tilde{Y}_t^N = \varphi_t^N Q_t^N\,, \qquad (\tilde{Y}_t^N) \tag{63}$$

$$(1+\kappa\, CCSR_t)\varphi_t^{CCS} h_t^{CCS} \geq \tau_t \perp CCSR_t \geq 0, \qquad (CCSR_t) \tag{47}$$

$$\varphi_t^j = \zeta_t^j + \eta_t^j\,, \qquad (Q_t^j, j{=}F,N,CCS) \tag{42}$$

$$\xi_t^j = b^j \eta_t^j\,, \qquad (\widetilde{M}_t^j, j{=}F,N,CCS) \tag{43}$$

$$1 = a^j \beta_t \zeta_{t+1}^j\,, \qquad (I_{t-1}^j, j{=}F,N,CCS) \tag{44}$$

$$\tilde{\xi}_t^j = (1-\delta)\beta_t \tilde{\xi}_{t+1}^j + \Omega_t^{\lambda}\,, \qquad (M_t^j, j{=}F,N,CCS) \tag{67}$$

Emissions and climate change:

Equation (18) calculates the total flow of carbon capture and storage, CCS_t, of both old and new vintages. In every period, the stock of stored carbon, S_t^{CCS}, increases with the flow of CCS_t, while a fixed share δ^{CCS} of the stored stock of carbon leaks into the atmosphere (19), and this leakage adds to future emissions (26).

$$EnEm_t = (1-\delta)EnEm_{t-1} + \varepsilon_t^F (1-CCSR_t)\tilde{Y}_t^F . \qquad (\tilde{\tau}_t) \qquad (17)$$

$$Em_t = (1-\delta)Em_{t-1} + \tilde{E}m_t . \qquad (\tilde{\tau}_t) \qquad (11)$$

$$ATM_{t+1} = (Em_t + \bar{\bar{E}}m_t + \delta^{CCS} S_{t-1}^{ccs}) + TR_{atm}^{atm} ATM_t + TR_{ul}^{atm} UL_t , \qquad (26)$$

$$UL_{t+1} = TR_{atm}^{ul} ATM_t + TR_{ul}^{ul} UL_t + TR_{ll}^{ul} LL_t , \qquad (27)$$

$$LL_{t+1} = TR_{ul}^{ll} UL_t + TR_{ll}^{ll} LL_t , \qquad (28)$$

$$F_t = 4.1^2 \ln(ATM_t / ATM_0) + EXOFORC_t , \qquad (29)$$

$$TEMP_{t+1} = TEMP_t + \delta^T (F_t / 4.1)(\bar{T} - TEMP_t) - TR_{TEMP}^{TLOW} (TEMP_t - TLOW_t) , \qquad (30)$$

Auxiliary budget condition (only in some cases):

$$\tilde{\tau}_t^j Em_t + tax_t^F \tilde{Y}_t^F = sub_t^N \tilde{Y}_t^F . \qquad (49)$$

We remark that the definition of welfare (2) is not necessary as part of the equilibrium equations when calculating the business as usual scenario, or a scenario with given exogenous carbon taxes. However, when calculating a scenario with a climate change stabilization target, the welfare equation serves to calculate the optimal timing of tax and subsidy rates, that is, carbon taxes can be calculated such that the climate change target is reached, and welfare is maximal given equilibrium conditions and this additional constraint.

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