C.I.R.E.D.
Centre
International
de Recherche
sur l'Environnement
et le Développement

IMACLIM-S

Commented formulary

February 5 2007

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version 1.1

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Introduction

This text provides a technical description of the aggregated one-region (global) two-good version of IMACLIM-S used in the TranSust project. The two goods are energy *E* and an aggregate of all other productions, or composite good, *Q*.

Notwithstanding such a high level of aggregation, IMACLIM-S is presented in a generalised *i*-sector format, to limit the number of equations to be commented and give a sense of how the composite sector is disaggregated in other versions of the model. The formulary is preceded by a short overview describing the purpose of the model and its general architecture.

I. IMACLIM-S: an overview

IMACLIM-S is a static general equilibrium framework designed to assess the macroeconomic impacts of a price- or quantity-based carbon policy at some medium to long term horizon. It focuses on the sensitivity of such impacts to the technical abatement costs and the particulars of the policy tools (tax reforms, trading systems, technical or emission standards) triggering abatement. It is built to allow for the description of economies with (*i*) a sub-optimal equilibrium on the labour market; (*ii*) behaviours of the underlying technical systems that cannot be approximated by 'well-behaved' production or utility functions for large departures from the reference scenario. The shifts of technical systems impact an aggregated induced technical change encompassing learning by doing and research and development activities; IMACLIM-S thus operates in an endogenous technical change framework.

Relying on the method of comparative-static analysis, it provides insights that are valid under the assumption that the transition from the reference growth path to a carbon-constrained equilibrated growth path is completed, and does not imply any transitory disequilibrium that would affect the derived equilibrium.

The main feature of IMACLIM-S is that assumptions regarding both the baseline projection of the energy system and its price-response at the projected

time horizon are harmonised with information from a bottom-up energy model, completed by a limited set of exogenous assumptions:

- The baseline projection to the time horizon is devised to be price- and quantity-consistent with a bottom-up projection of the energy system, based on shared assumptions regarding main economic and demographic drivers;
- At the time horizon, the response of the IO coefficients and of consumers to new relative prices is captured through envelops of the functions generated by various carbon prices, as reconstructed from bottom-up information. This method amounts to capturing the bias in technical change induced by the carbon constraint through a set of varying point elasticities. As opposed to conventional production specifications, the use of envelop functions allows for representing technical asymptotes and technological breakthroughs.[1]

In order to capture the *rate* and not only *direction* of technical progress, IMACLIM-S carries out a comparative-static analysis of an endogenous growth mechanism: investment triggers a Hicks-neutral technical progress affecting factors productivity. Crowding-out effects of biasing the direction of technical change are computed by withdrawing the investment dedicated to carbon abatement from total investment (under the assumption of a constant saving ratio), modulo spill over assumptions.

The most extensive calibrated version of IMACLIM-S to date pictures 14 world regions and three goods (composite, transformed energy, fossil fuels); for more in depth scrutiny of issues such as the role of fiscal reforms, competitiveness of exposed carbon intensive industry or distributive effects, a 9-sector 2-region (country or region analysed and rest of the world) version is preferred.

II. Technical description

The following technical description presents the one-region two-good version of IMACLIM-S used during the TranSust project. It entails a nomenclature of

[1] For an extensive presentation of the hybridising methodology behind these envelops cf. Ghersi F. and J.-C. Hourcade (2006), "Macroeconomic Consistency Issues in E3 Modeling: The Continued Fable of the Elephant and the Rabbit", *The Energy Journal*, Hybrid Modeling of Energy-Environment Policies: Reconciling Bottom-Up and Top-Down: 39-62.

notations and a formulary of 28 equations, completed by a brief comment on each of the latter.

II.1. Model nomenclature

As a comparative-static model, IMACLIM consists in a system of equations:

$$\begin{cases} f_1(x_1,\ldots,x_n, z_1,\ldots,z_m) = 0 \\ f_2(x_1,\ldots,x_n, z_1,\ldots,z_m) = 0 \\ \ldots \\ f_n(x_1,\ldots,x_n, z_1,\ldots,z_m) = 0 \end{cases}$$

with

- x_i, $i \in [1, n]$, a set of variables (as many as equations),
- z_i, $i \in [1, m]$, $m < n$, a set of parameters,
- f_i, $i \in [1, n]$, a set of functions, some of them non-linear in x_i.

Calibration consists in defining a set of $x_1,\ldots, x_n$ and solving the system for $z_1,\ldots, z_m$. In the case of TranSust simulations, the x vector used for the calibration of IMACLIM-S—its business as usual (BAU) equilibrium—is based on a global projection of GTAP 5.0's 1997 accounting matrices.[2] In more recent exercises, it is derived from a harmonised reference projection of the IMACLIM-R (a dynamic recursive version of IMACLIM) and the POLES model of global energy systems.[3] In both instances, the economic equilibria expressed in monetary flows are disaggregated in prices and quantities by fixing the vector of prices, thereby defining quasi-units for all goods (including labour), without loss of generality.

A limited set of parameters is exogenous to the calibration, based either on econometric specifications, or on mere assumptions (control variables).

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[2] This projection is again a comparative-static exercise, based on a simplified version of IMACLIM, IMACLIM-PROJTES, and grounded on an evolution of the energy systems as projected by the POLES model (with which it shares the two central assumptions of GDP and population growth).

[3] Cf. Criqui P. (2001), *POLES: Prospective Outlook on Long-term Energy Systems*, Institut d'Économie et de Politique de l'Énergie, Grenoble, France, 9 pp. http://www.upmf-grenoble.fr/iepe/textes/POLES8p_01.pdf (accessed August 2006)

The following presentation of the notations used in the model is divided in the three categories outlined above: variables, calibrated parameters, exogenous parameters and control variables. Within each category the notations are sorted in alphabetical order.[4]

II.1.1. Variables (x vector)

a_{EE} Energy intensity of energy (unit intermediate consumption of energy E in the production of energy E, input-output coefficient).

a_{EQ} Energy intensity of the composite good (unit intermediate consumption of energy E in the production of the composite good Q, input-output coefficient).

C_i Household consumption of good i (in real terms).

g_{Ci} Carbon (or CO_2, depending on calibration) emitted per unit of final consumption of good i.

g_{ij} Carbon (or CO_2, cf. *supra*) emitted per unit of consumption of good i in the production of good j.

IPC Consumer price index.

k_{AQ} Carbon tax-induced investment per unit of production of good i (in real terms). Nil per definition in the BAU projection.

k_E Unit fixed capital consumption in the production of energy E (in real terms).

K_i Consumption of good i in fixed capital formation (in real terms). In the two-sector version of the model, K_E is irrelevant and K_i boils down to K_Q.

l_Q Labour intensity of the composite good (unit labour consumption in the production of good Q, in real terms).

p_{Ci} Consumer price of good i, household consumption.

p_{Gi} Consumer price of good i, public consumption.

F_Q Endogenous technical progress coefficient applying to the composite production. Normalised in the BAU projection (cf. *infra*). Energy pro-

[4] The Greek letters are sorted following their English denomination rather than their Latin equivalent.

duction is not corrected by a similar coefficient: it is assumed that the more detailed BU expertise providing the basis of energy technology shifts (endogenous variations of the a_{EE} and k_E coefficients) takes into account all sources of factor productivity variations.

p_K Capital price index (weighted average of the price of capital goods). With K_Q the only relevant component of fixed capital formation p_K boils down to p_{KQ}.

p_{Ki} Price of good i used in fixed capital formation.

p_{Li} Unit labour cost in the production of good i.

p_{Yi} Producer price of good i.

R Household income (gross disposable income).

R_T Transfer income.

s_i Payroll tax rate in the production of good i.

T Total tax and social security contributions.

Q_Q Decreasing returns coefficient applying to composite production. Normalised in the BAU projection. Energy production is not corrected by a similar coefficient (cf. F_Q above).

u Unemployment rate.

w_E Net wage in the production of energy.

Y_i Output of good i (in real terms).

II.1.2. Calibrated parameters

a_{QQ} Composite intensity of the composite good (unit consumption of the composite good Q in its own production, input-output coefficient).

a_{QE} Composite intensity of the energy good (unit consumption of the composite good Q in the production of energy E, input-output coefficient). The degree of approximation stemming from fixing a_{QQ} and a_{QE} is limited by the highly aggregated nature of good Q.

G_i Public (government) consumption of good i (in real terms). Is held constant (i) for lack of behavioural assumptions regarding governments, and (ii) as this makes the shifts in final consumption a better qualitative indicator of welfare variations.

k_Q Unit fixed capital consumption in the production of the composite good Q (in real terms). The baseline data, unavailable from GTAP, is calibrated so that the difference between total investment and households' savings should equal total fixed capital consumption (in monetary terms).

L Labour supply (unit defined by the numéraire w_Q, cf. *infra*).

l_E Labour intensity of energy (unit consumption of labour in the production of energy E). Is held constant, considering both its relatively small weight in the energy cost structure, and impact on the aggregate labour market.

p_i Mark-up rate in the production of good i.

r_C Households' saving rate.

r_T Ratio of fiscal contributions to total output. A constant r_T applies only in the payroll tax-recycling version of the model, where it defines the revenue neutrality of the fiscal reform accompanying the carbon policy.

t_{Ci} Tax rate on the sales of good i to households.

t_{Gi} Tax rate on the sales of good i to public agents.

t_i Output tax rate (on the production of good i).

t_{IR} Income tax rate.

t_{IS} Corporate tax rate.

t_{ji} Tax rate on the sales of good j entering the production of good i.

t_{Ki} Tax rate on the sales of good i to form fixed capital.

II.1.3. Econometrically estimated parameters and control variables

e_{FK} Elasticity of the technical progress coefficient F_Q to the evolution of total fixed capital consumption in the composite sector. The latter argument is intended as a proxy of the investment differential cumulated between the BAU and the policy-induced growth path. An econometric analysis made for France has not been extended to other regions yet. For the time being, a conservative learning curve assumption is used for all sectors: a doubling of real investment leads to a 10% decrease in factor consumption. This assumption corresponds to a .15 value for e_{FK}.

e_{wu} Elasticity of the real wage to the unemployment rate. Set to 0.1 following a seemingly robust empirical estimate[5].

t_{ev} Crowding-out assumption: share of the investment dedicated to carbon abatement that is subtracted from general productivity investment (cf. *infra*). Sensitivity analysis reveals a paramount importance of the choice of t_{ev}.

t_C Tax per unit of carbon as defined by g_{ij} and g_{Ci}. The main control variable in the model.

w_Q Net wage in the production of the composite good Q. Following common practice, w_Q is exogenously set in the calibration process to disaggregate prices and quantities on the labour market (thereby defining a unit for labour *L*, without loss of generality). With labour used as the numéraire it is then kept constant.[6]

II.2. Formulary

In the following equations, notations with a '0' subscript stand for values in the BAU projection.

II.2.1. Prices

$$p_{Yi} = \frac{Q_i}{F_i}\left(\sum_{j=E,Q} (p_{Yj} + t_C\, g_{ji})\,(1+t_{ji})\, a_{ji} + p_{Li}\, l_i + p_K\, k_i \right) + t_i\, p_{Yi} + p_{Yi}\, p_i \qquad (1)$$

$$p_{Ci} = (p_{Yi} + t_C\, g_{Ci})\,(1+t_{Ci}) \qquad (2)$$

$$p_{Gi} = p_{Yi}\,(1+t_{Gi}) \qquad (3)$$

$$p_{Ki} = p_{Yi}\,(1+t_{Ki}) \qquad (4)$$

$$IPC = \frac{\sum_{i=E,Q} p_{Ci}\, C_{i0}}{\sum_{i=E,Q} p_{Ci0}\, C_{i0}} \qquad (5)$$

[5] Cf. Blanchflower D. G. and A. J. Oswald (1995). "An Introduction to the Wage Curve". *The Journal of Economic Perspectives* 9 (3): 153-167.

[6] The model is conventionally homothetic in prices, and a numéraire must be defined in which all prices can be measured. With labour the numéraire, prices fluctuation are intuitively interpretable as the variations of the quantity of work necessary to buy one good.

$$p_K = \frac{\sum_{i=E,Q} p_{Ki}\, K_{i0}}{\sum_{i=E,Q} p_{Ki0}\, K_{i0}}\, p_{K0} \tag{6}$$

$$p_{Li} = w_i\,(1+s_i) \tag{7}$$

II.2.2. Households

$$R = (1-t_{IR}) \sum_{i=E,Q} w_i \frac{Q_i}{F_i} l_i\, Y_i + (1-t_{IS}) \sum_{i=E,Q} p_{Yi}\, p_i\, Y_i + R_T \tag{8}$$

$$r_C\, R = \sum_{i=E,Q} p_{Ci}\, C_i \tag{9}$$

$$\frac{p_{CE}\, C_E}{r_C\, R} = f_1\left(\frac{p_{CE}}{p_{CQ}}\right) \tag{10}$$

II.2.3. Government

$$\begin{aligned} T = {} & \sum_{j=E,Q} \sum_{i=E,Q} t_{ij}\,(p_{Yi} + t_C\, g_{ij}) \frac{Q_j}{F_j} a_{ij} + \sum_{i=E,Q} s_i\, w_i \frac{Q_i}{F_i} l_i\, Y_i + \sum_{i=E,Q} t_i\, p_{Yi}\, Y_i \\ & + t_{IR} \sum_{i=E,Q} w_i \frac{Q_i}{F_i} l_i\, Y_i + t_{IS} \sum_{i=E,Q} p_{Yi}\, p_i\, Y_i + \sum_{i=E,Q} t_{Ci}\, p_{Yi}\, C_i + \sum_{i=E,Q} t_{Gi}\, p_{Yi}\, G_i \\ & + \sum_{i=E,Q} t_{Ki}\, p_{Yi}\, K_i + t_C \left(\sum_{j=E,Q} \sum_{i=E,Q} g_{ij} \frac{Q_j}{F_j} a_{ij}\, Y_j + \sum_{i=E,Q} g_{Ci}\, C_i \right) \end{aligned} \tag{11}$$

$$T = \sum_{i=E,Q} p_{Gi}\, G_i + R_T \tag{12}$$

$$T = r_T \sum_{i=E,Q} p_{Yi}\, Y_i \tag{13}$$

$$s_i = s_{i0} \frac{s_Q}{s_{Q0}} \tag{14}$$

II.2.4. Investment market

$$\sum_{i=E,Q} p_{Ki}\, K_i = p_K \sum_{i=E,Q} \frac{Q_i}{F_i} k_i\, Y_i + (1-r_C)\, R \tag{15}$$

II.2.5. Labour market

$$L = \sum_{i=E,Q} \frac{Q_i}{F_i} l_i\, Y_i + u\, L \tag{16}$$

$$\frac{w_Q}{IPC} u^{e_{wu}} = w_{Q0}\ u_0^{e_{wu}} \tag{17}$$

II.2.6. Goods market

$$Y_i = \sum_{j=\mathrm{E,Q}} \frac{Q_j}{F_j} a_{ji}\ Y_j + C_i + G_i + K_i \tag{18}$$

II.2.7. Technology biases

$$k_Q = k_{Q0} + (1\text{-}t_{ev})\ k_{AQ} \tag{19}$$

$$k_{AQ} = f_2\left(\frac{t_C}{IPC}\right) \tag{20}$$

$$k_E = k_{E0}\ f_3\left(\frac{t_C}{IPC}\right) \tag{21}$$

$$a_{EQ} = a_{EQ0}\ f_3\left(\frac{(p_{YE} + t_C\ g_{EQ})\ (1+t_{EQ})}{p_{LQ}}\right) \tag{22}$$

$$\frac{a_{EQ}}{l_Q} = \frac{a_{EQ0}}{l_{Q0}}\ f_4\left(\frac{(p_{YE} + t_C\ g_{EQ})\ (1+t_{EQ})}{p_{LQ}}\right) \tag{23}$$

$$a_{EE} = a_{EE0}\ f_5\left(\frac{(p_{YE} + t_C\ g_{EE})\ (1+t_{EE})}{IPC}\right) \tag{24}$$

II.2.8. Carbon emission coefficients

$$g_{Ci} = f_6\left(\frac{t_C}{IPC}\right) \tag{25}$$

$$g_{ij} = f_7\left(\frac{t_C}{IPC}\right) \tag{26}$$

II.2.9. Productivity factors

$$F_Q = \left(\frac{k_{Q0}\ Y_Q - t_{év}\ k_{AQ}\ Y_Q}{k_{Q0}\ Y_{Q0}}\right)^{e_{FKQ}} \tag{27}$$

$$Q_Q = \left(\frac{Y_Q}{Y_{Q0}}\right)^{\frac{p_Q}{1-p_Q}} \tag{28}$$

II.3. Comments

(1) Producer prices are defined according to the cost structure of production, that is the sum of intermediate consumption, labour costs, capital amortisement, an output tax and a relative mark-up. Input-output coefficients a_{ji}, labour consumption l_i and capital amortisement k_i are (*i*) multiplied by a decreasing returns coefficient Q_i and (*ii*) divided by a technical progress coefficient F_i.[7] The underlying theoretical stance is that the quantity—together with the substitutability and eventually the balance—of physical factors consumptions in the production are linked to the particular capital structure induced by the carbon policy under scrutiny.

(2-4) Consumer prices equal the producer prices plus taxes, including, for the consumption of households,[8] a tax on the related carbon emissions.

(5) Consumer price index (Laspeyres defined).

(6) Investment price index. Irrelevant in the two-sector version of the model, where the composite good Q is the only eligible good for capital formation.

(7) Labour costs equal net wages plus *ad valorem* payroll taxes. Contributions from the employer and the employee are not distinguished.

(8) Households' income R equals revenues from labour (wages net of payroll taxes and income tax) and profits (net of corporate tax) plus public transfers R_T.

(9) Households' budget constraint: households' consumption amounts to a constant share r_C of total income.

(10) The households' budget share of energy expenditures is a function f_1 of the energy price relative to the price of the competing composite

[7] As previously indicated the two coefficients impact the composite cost structure only (i.e. $Q_E = F_E = 1$).

[8] Neither investment nor public expenditures consume any energy goods in national accounting systems—public expenditures are concentrated on a single 'public good' produced by a public sector with its own cost structure.

good. f_1 is an envelop of the point demand functions induced by increasing price signals, calibrated on bottom-up results.[9]

(11) Public revenues accrue from taxes on intermediate consumptions, payroll taxes, output taxes, the income and corporate taxes, final consumption taxes, public consumption taxes, investment taxes, and the carbon tax (levied on the carbon emissions related to the intermediate and final consumptions).

(12) Government's budget constraint: public income equals public consumption plus transfers to the households.

(13) Revenue neutrality assumption: r_T the ratio of total public revenues over total nominal output is held constant (at its calibrated value); another possible assumption, with non-negligible impacts on the simulation results, is a constancy of the public revenue to GDP ratio.

(14) Payroll tax rates evolve identically in all sectors.

(15) Balance on the investment market: total investment equals the sum of total fixed capital consumption and of a constant share of households' revenues (constant saving ratio).

(16) Balance on the labour market: total labour equals the sum of employed and unemployed labour. Not detailing this equilibrium for every production amounts to the implicit assumption of 'mobile'—not sector-specific—labour.[10]

(17) Wage curve: real wages in the production of good Q are inversely correlated to unemployment. Resorting to a wage curve allows for picturing unemployment, which is of particular interest when analysing specific carbon-levy recycling schemes.

(18) Balance on good i's market: total output equals the sum of intermediate consumption, household's consumption, public consumption and investment.

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[9] Again, cf. Ghersi F. and J.-C. Hourcade (2006), "Macroeconomic Consistency Issues in E3 Modeling: The Continued Fable of the Elephant and the Rabbit", *The Energy Journal*, Hybrid Modeling of Energy-Environment Policies: Reconciling Bottom-Up and Top-Down: 39-62.

[10] The assumption is arguably reasonable, bearing in mind the comparative-static nature of IMACLIM simulation: the alternate growth path leading to the derived equilibrium at a mid- to long-term horizon allows for qualitative labour adjustments.

(19) Real fixed capital consumption in the production of good Q is equal to its BAU value plus a share of the amortisement of the abatement-specific unitary investment, modulo the extent of crowding-out effects: outside such effects (t_{ev} = 0) abatement-dedicated investments enter the cost structure on top of BAU investments (cost increases are entirely passed through to consumers); if crowding-out occurs to some extent (0 < t_{ev} = 1) they are partly substituted to BAU investments.

(20) Unitary investment linked to abatement-dedicated capital in the production of good Q is a function f_2 of the IPC-corrected carbon tax. f_2 is calibrated on bottom-up results.

(21) Capital intensity of energy (real fixed capital consumption in the production of good E) is a function f_3 of the IPC-corrected carbon tax. f_1 is an envelop of the point factor demand functions induced by increasing price signals, calibrated on bottom-up results.

(22-23) The energy and labour intensities of the composite good are functions of the corresponding relative prices. f_3 and f_4 are again envelop specifications built on bottom-up results. As stated in the commentary to equation (1), the underlying theoretical viewpoint is that any carbon policy primarily induces changes in the structure of capital, resulting in a 'capital vintage' eventually characterised by a trade-off between the remaining factors—trade-off grounded on an envelop function of the results from an associated bottom-up model.

(24) Energy intensity of energy (real energy consumption in the production of good E) is a function f_5 of the relative price of energy. f_5 is an envelop of point factor demand functions calibrated on bottom-up results.

(26) g_{Ci} is a function of t_C as drawn from the results of an associated bottom-up model, accounting for implicit fuel substitution in the composition of final fossil fuel and transformed energy demands.

(26) Same as g_{Ci}, it is a function of t_C as drawn from the results of an associated bottom-up model.

(27) Induced technical change coefficient in the production of good *Q*—see comment (1). The amount of real investment positively correlated to a rise in productivity (e_{fKQ} > 0) is computed net of the share

of the abatement-specific investment substituted to, rather than added up to, general-productivity investment.

(28) Decreasing returns in the production of good Q are a function of Y_Q dictated by analytical calculus on equation (1) to allow for marginal cost pricing. They cause an exponential rise in factor consumptions as production supersedes its baseline level—which is thus used as a reference level for all alternative growth scenarios at the time horizon explored.